Całki nieoznaczone

Zadania

Zadanie 9.1. Korzystając ze wzorów na całki elementarne oblicz:

(a) $\int \left(x^3 - 2x^2 + 5 + \dfrac{1}{x} \right) \mathrm{d}x;$

(b) $\int \dfrac{10x^8 + 3}{x^4} \mathrm{d}x;$

(c) $\int \left(\dfrac{1}{\sqrt{x}} - \dfrac{1}{\sqrt[4]{x^3}} \right) \mathrm{d}x;$

(d) $\int \mathrm{e}^x \left(1 - \dfrac{\mathrm{e}^{-x}}{x^2} \right) \mathrm{d}x;$

(e) $\int \dfrac{\cos 2x}{\sin^2 x \cos^2 x} \mathrm{d}x;$

(f) $\int \dfrac{\mathrm{d}x}{\sin^2 x \cos^2 x};$

(g) $\int \sin^2 \left(\dfrac{1}{2} x \right) \mathrm{d}x.$

(h) $\int \mathrm{ctg}^2 x \, \mathrm{d}x.$

Zadanie 9.2. Korzystając ze wzorów na całkowanie przez części oraz przez podstawienie oblicz:

(a) $\int \sin \dfrac{x}{2} \mathrm{d}x;$

(b) $\int \mathrm{e}^{-3x} \mathrm{d}x;$

(c) $\int \dfrac{\mathrm{d}x}{\cos^2 5x};$

(d) $\int (3 - 2x)^5 \, \mathrm{d}x;$

(e) $\int x \ln x \, \mathrm{d}x$

(f) $\int \ln x \, \mathrm{d}x;$

(g) $\int \dfrac{\ln x}{x} \mathrm{d}x;$

(h) $\int \dfrac{x}{x^2 + 1} \mathrm{d}x;$

(i) $\int x^2 \cos x \, \mathrm{d}x;$

(j) $\int \mathrm{ctg}\, x \, \mathrm{d}x;$

(k) $\int x \arctan x \, \mathrm{d}x;$

(l) $\int \dfrac{\cos 2x}{\sin x \cos x} \mathrm{d}x;$

(m) $\int \dfrac{\ln x}{x^2} \mathrm{d}x;$

(n) $\int \dfrac{e^{\frac{1}{x}}}{x^2} \mathrm{d}x;$

(o) $\int x^3 e^{-x} \mathrm{d}x;$

(p) $\int e^x \sin x \, \mathrm{d}x;$

(q) $\int \arctan x \, \mathrm{d}x.$

Zadanie 9.3. Sprowadzając następujące całki do całek postaci $\int \dfrac{\mathrm{d}x}{1+x^2}$ oraz $\int \dfrac{\mathrm{d}x}{\sqrt{1-x^2}}$ oblicz:

(a) $\int \dfrac{\mathrm{d}x}{9+x^2}$;

(b) $\int \dfrac{\mathrm{d}x}{\sqrt{4-x^2}}$;

(c) $\int \dfrac{x^2}{x^6+4}\mathrm{d}x$;

(d) $\int \dfrac{x}{\sqrt{3-x^4}}\mathrm{d}x$;

(e) $\int \dfrac{5x-2}{x^2+4}\mathrm{d}x$;

(f) $\int \dfrac{x^2}{x^2+1}\mathrm{d}x$;

(g) $\int \dfrac{\mathrm{d}x}{x^2+4x+5}$;

(h) $\int \dfrac{\mathrm{d}x}{\sqrt{3-2x-x^2}}$;

(i) $\int \dfrac{\mathrm{d}x}{\sqrt{4x-x^2}}$.

Zadanie 9.4. Oblicz następujące całki funkcji wymiernych:

(a) $\int \dfrac{\mathrm{d}x}{x^2-25}$;

(b) $\int \dfrac{x^3}{x^8-1}\mathrm{d}x$;

(c) $\int \dfrac{x^3}{x-2}\mathrm{d}x$;

(d) $\int \dfrac{x^4}{x^2+9}\mathrm{d}x$;

(e) $\int \dfrac{x^5}{x^3-8}\mathrm{d}x$;

(f) $\int \dfrac{x-4}{(x-2)(x-3)}\mathrm{d}x$;

(g) $\int \dfrac{2x+7}{x^2+x-2}\mathrm{d}x$;

(h) $\int \dfrac{(x+1)^3}{x^2-x}\mathrm{d}x$;

(i) $\int \dfrac{2x^2-5x+1}{x^3-2x^2+x}\mathrm{d}x$;

(j) $\int \dfrac{2x^2+x+4}{x^3+x^2+4x+4}\mathrm{d}x$;

(k) $\int \dfrac{3x^2+2x+1}{(x+1)^2(x^2+1)}\mathrm{d}x$;

(l) $\int \dfrac{\mathrm{d}x}{x^2-2x}$;

(m) $\int \dfrac{\mathrm{d}x}{x^4-x^2}$;

(n) $\int \dfrac{\mathrm{d}x}{x^4-1}$.

Zadanie 9.5. Oblicz następujące całki trygonometryczne:

(a) $\int \sin^2(3x)\mathrm{d}x$;

(b) $\int (1+2\cos x)^2\mathrm{d}x$;

(c) $\int \sin^2 x \cos^3 x\mathrm{d}x$;

(d) $\int \sin^2 x \cos^2 x\mathrm{d}x$;

(e) $\int \sin^5 x\mathrm{d}x$;

(f) $\int \cos^4 x\mathrm{d}x$;

(g) $\int \dfrac{\cos^3 x}{\sin^2 x}\mathrm{d}x$;

(h) $\int \dfrac{\mathrm{d}x}{\sin(2x)}$;

(i) $\int \sin(3x)\cos x\mathrm{d}x$;

(j) $\int \cos(5x)\cos(7x)\mathrm{d}x$;

(k) $\int \dfrac{2+\sin x}{\sin x(1+\cos x)}\mathrm{d}x$;

(l) $\int \dfrac{\cos x}{1+\cos x}\mathrm{d}x$.

Zadanie 9.6. Oblicz następujące całki:

(a) $\int \dfrac{\mathrm{d}x}{\sqrt{x}+\sqrt[3]{x}}$;

(b) $\int \dfrac{1+\sqrt{x}}{1-\sqrt{x}}\mathrm{d}x$;

(c) $\int x\sqrt{x+5}\mathrm{d}x$;

(d) $\int \dfrac{\mathrm{d}x}{\sqrt[3]{7-\frac{1}{2}x}}$;

(e) $\int \dfrac{\mathrm{d}x}{\sqrt{x^2+k}}$;

(f) $\int \dfrac{\mathrm{d}x}{\sqrt{x^2-6x+15}}$;

(g) $\int \sqrt{x^2+k}\mathrm{d}x$;

(h) $\int \sqrt{x^2-2x+5}\mathrm{d}x$;

(i) $\int \sqrt{2x+x^2}\mathrm{d}x$.

Zadanie 9.1. (a) $\frac{1}{4}x^4 - \frac{2}{3}x^3 + 5x + \ln|x| + C$; (b) $2x^5 - x^{-3} + C$; (c) $2x^{\frac{1}{2}} - 4x^{\frac{1}{4}} + C$; (d) $e^x + \frac{1}{x} + C$;
(e) $-\operatorname{ctg} x - \operatorname{tg} x + C$; (f) $\operatorname{tg} x - \operatorname{ctg} x + C$; (g) $\frac{1}{2}x - \frac{1}{2}\sin x + C$; (h) $-\operatorname{ctg} x - x + C$.

Zadanie 9.2. (a) $-2\cos\frac{x}{2} + C$; (b) $-\frac{1}{3}e^{-3x} + C$; (c) $\frac{1}{5}\operatorname{tg} 5x + C$; (d) $-\frac{1}{12}(3 - 2x)^6 + C$;
(e) $\frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$; (f) $x\ln x - x + C$; (g) $\frac{1}{2}\ln^2 x + C$; (h) $\frac{1}{2}\ln(x^2 + 1) + C$; (i) $x^2\sin x + 2x\cos x$
$-2\sin x + C$; (j) $\ln|\sin x| + C$; (k) $\frac{1}{2}x^2\operatorname{arctg} x - \frac{1}{2}x + \frac{1}{2}\operatorname{arctg} x + C$; (l) $\ln|\sin x| + \ln|\cos x| + C$;
(m) $-\frac{1}{x}\ln x - \frac{1}{x} + C$; (n) $-e^{\frac{1}{x}} + C$; (o) $-x^3e^{-x} - 3x^2e^{-x} - 6xe^{-x} - 6e^{-x} + C$; (p) $\frac{1}{2}e^x(\sin x - \cos x) + C$;
(q) $x\operatorname{arctg} x - \frac{1}{2}\ln(1 + x^2) + C$.

Zadanie 9.3. (a) $\frac{1}{3}\operatorname{arctg}\frac{x}{3} + C$; (b) $\arcsin\frac{x}{2} + C$; (c) $\frac{1}{6}\operatorname{arctg}\frac{x^3}{2} + C$; (d) $\frac{1}{2}\arcsin\frac{x^2}{\sqrt{3}} + C$;
(e) $\frac{5}{2}\ln(x^2 + 4) - \operatorname{arctg}\frac{x}{2} + C$; (f) $x - \operatorname{arctg} x + C$; (g) $\operatorname{arctg}(x + 2) + C$; (h) $\arcsin\frac{x+1}{2} + C$;
(i) $\arcsin\frac{x-2}{2} + C$;

Zadanie 9.4. (a) $\frac{1}{10}\ln\left|\frac{x-5}{x+5}\right| + C$; (b) $\frac{1}{8}\ln\left|\frac{x^4-1}{x^4+1}\right| + C$; (c) $\frac{1}{3}x^3 + x^2 + 4x + 8\ln|x - 2| + C$;
(d) $\frac{1}{3}x^3 - 9x + 27\operatorname{arctg}\frac{x}{3} + C$; (e) $\frac{1}{3}x^3 + \frac{8}{3}\ln|x^3 - 8| + C$; (f) $\ln\frac{(x-2)^2}{|x-3|} + C$; (g) $\ln\frac{|x-1|^3}{|x+2|} + C$;
(h) $\frac{1}{2}x^2 + 4x + \ln\frac{|x-1|^8}{|x|} + C$; (i) $\ln|x(x-1)| + \frac{2}{x-1} + C$; (j) $\ln\left(|x+1|\sqrt{x^2 + 4}\right) + C$; (k) $\ln\frac{\sqrt{x^2+1}}{|x+1|} - \frac{1}{x+1}$
$+ \operatorname{arctg} x + C$; (l) $\frac{1}{2}\ln\left|\frac{x-2}{x}\right| + C$; (m) $\frac{1}{x} + \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C$; (n) $\frac{1}{4}\ln\left|\frac{x-1}{x+1}\right| - \frac{1}{2}\operatorname{arctg} x + C$.

Zadanie 9.5. (a) $\frac{1}{2}x - \frac{1}{12}\sin 6x + C$; (b) $3x + 4\sin x + \sin 2x + C$; (c) $\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C$;
(d) $\frac{1}{8}x - \frac{1}{32}\sin 4x + C$; (e) $-\sin^4 x \cos x - \frac{4}{3}\cos^3 x + \frac{4}{5}\cos^5 x + C$; (f) $\sin x \cos^3 x - \frac{3}{32}\sin 4x + \frac{3}{8}x + C$;
(g) $-\sin x - \frac{1}{\sin x} + C$; (h) $\frac{1}{2}\ln|\operatorname{tg} x| + C$; (i) $-\frac{1}{8}\cos 4x - \frac{1}{4}\cos 2x + C$; (j) $\frac{1}{24}\sin 12x + \frac{1}{4}\sin 2x + C$;
(k) $\frac{1}{2}\operatorname{tg}^2\frac{x}{2} + \operatorname{tg}\frac{x}{2} + \ln|\operatorname{tg}\frac{x}{2}| + C$; (l) $x - \operatorname{tg}\frac{x}{2} + C$.

Zadanie 9.6. (a) $2\sqrt{x} - 3\sqrt[3]{x} + \sqrt[6]{x} - 6\ln(1 + \sqrt[6]{x}) + C$; (b) $-x - 4\sqrt{x} - 4\ln|1 - \sqrt{x}| + C$;
(c) $\frac{2}{5}(x+5)^{\frac{5}{2}} - \frac{10}{3}(x+5)^{\frac{3}{2}} + C$; (d) $-3(7 - \frac{1}{2}x)^{\frac{2}{3}} + C$; (e) $\ln|x + \sqrt{x^2 + k}| + C$; (f) $\ln|x - 3 + \sqrt{x^2 - 6x + 15}|$
$+ C$; (g) $\frac{1}{2}x\sqrt{x^2 + k} + \frac{1}{2}k\ln|x + \sqrt{x^2 + k}| + C$; (h) $2\ln|x - 1 + \sqrt{x^2 - 2x + 5}| + \frac{1}{2}(x-1)\sqrt{x^2 - 2x + 5} + C$;
(i) $\frac{1}{2}(x + 1)\sqrt{2x + x^2} - \frac{1}{2}\ln|x + 1 + \sqrt{2x + x^2}| + C$.