Uniwersytet im. Adama Mickiewicza w Poznaniu
Wydział Matematyki i Informatyki

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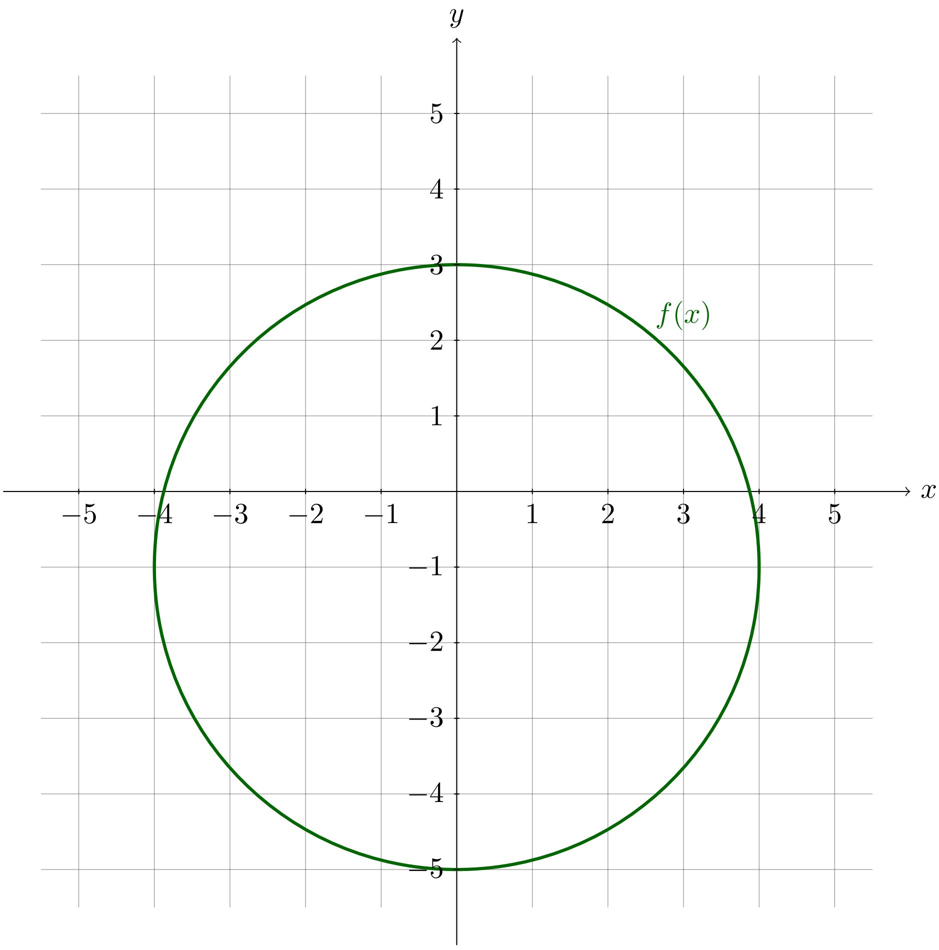
Ordinary differential equations figures

Poznań 2012

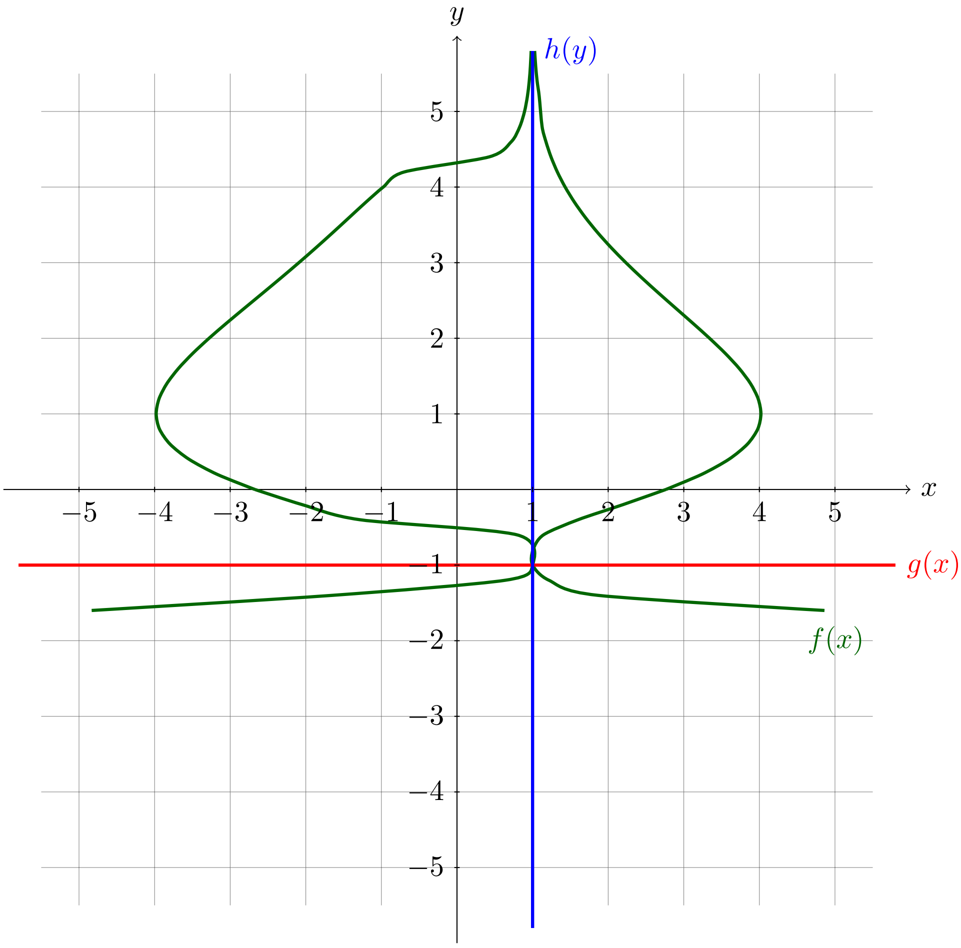
Equation: $x\mathrm{d}x + (y+1)\mathrm{d}y = 0$



Solutions

General: The function $f(x)$ is such that $x^2+(f(x)+1)^2 = C$, where C is a constant (here $C = 4$).

Equation: $x^2(1+y)\mathrm{d}x + (x^3-1)(y-1)\mathrm{d}y = 0$

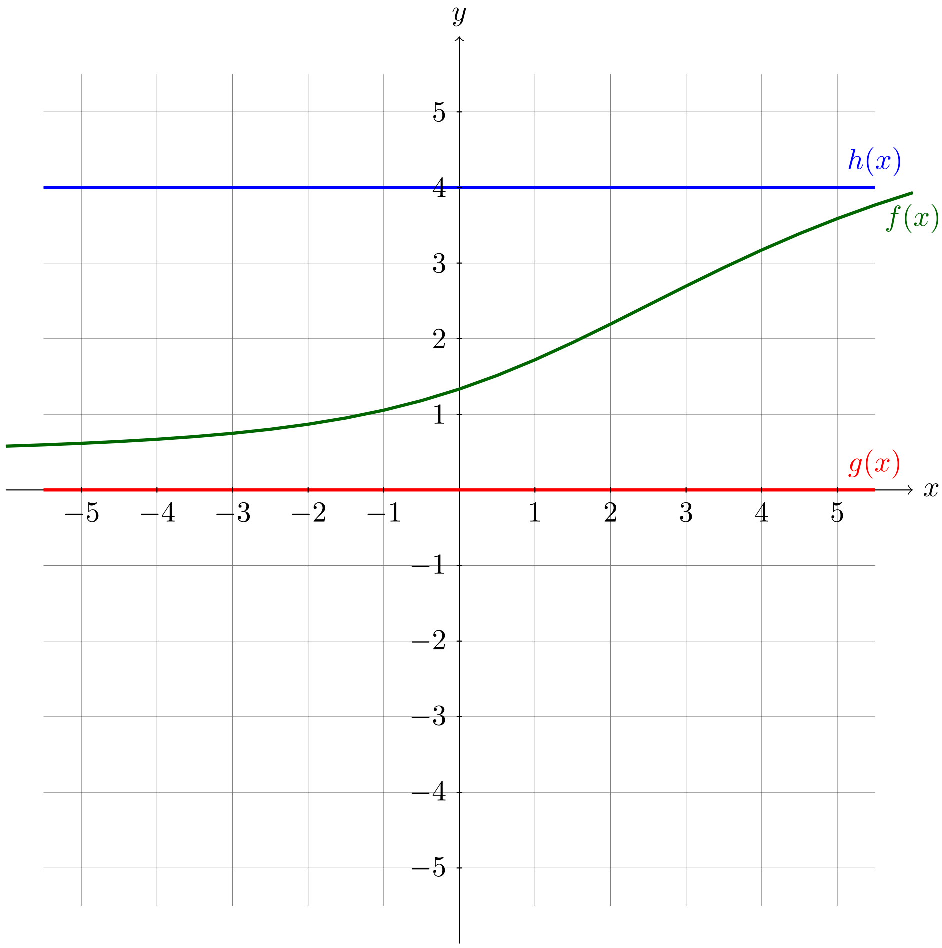


Solutions

General: $f(x)$ is such that $\frac{1}{3}\ln\left|x^3-1\right| + f(x) - 2\ln|f(x)+1| = C$ (here $C=1$)

Particular: $g(x) = -1$ for $x \neq 1$,
$h(y) = 1$ for $y \neq -1$.

Equation: $2y\sqrt{by-y^2}\mathrm{d}x-(b^2+x^2)\mathrm{d}y=0$

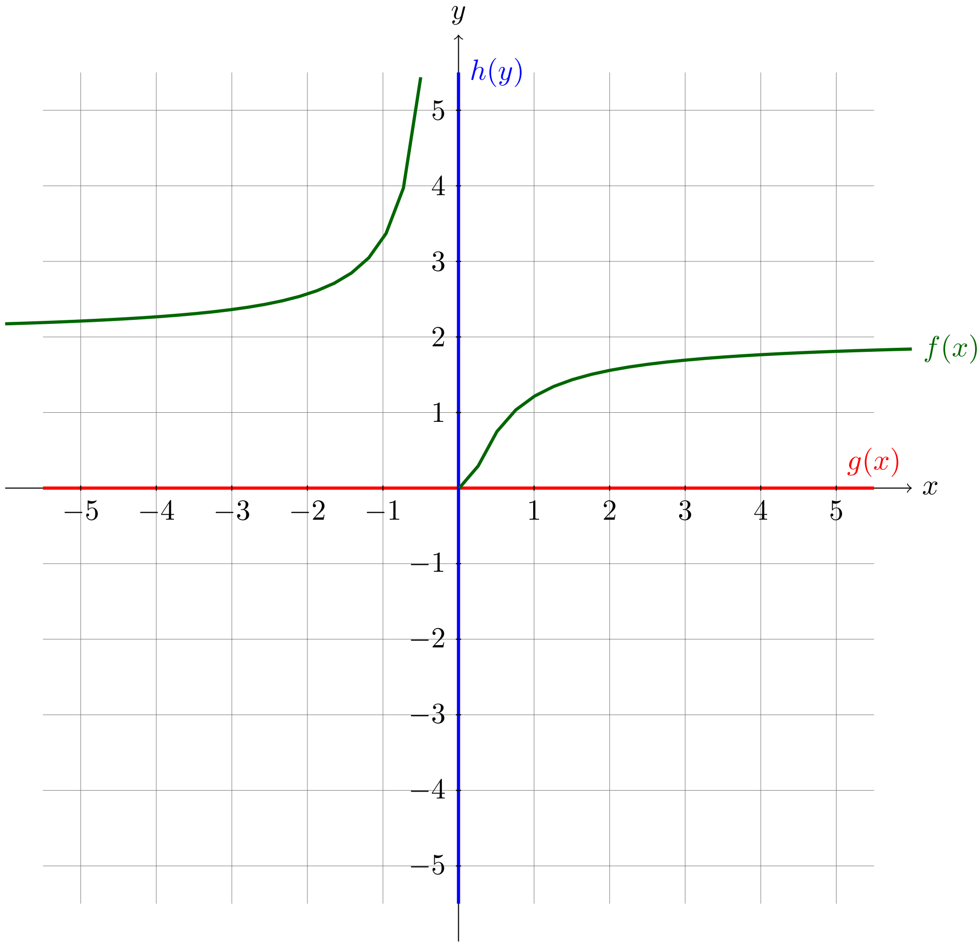


Solutions

General: $f(x)=\frac{b}{\left(C-\arctan\frac{x}{b}\right)^2+1}$ for x such that $C-\arctan\frac{x}{b}>0$ (here $b=4$ and $C=2$, so $x\in\mathbb{R}$),

Particular: $g(x)=0$,
$h(x)=b$ (singular solution).

Equation: $2x^2\frac{\mathrm{d}y}{\mathrm{d}x} = y$

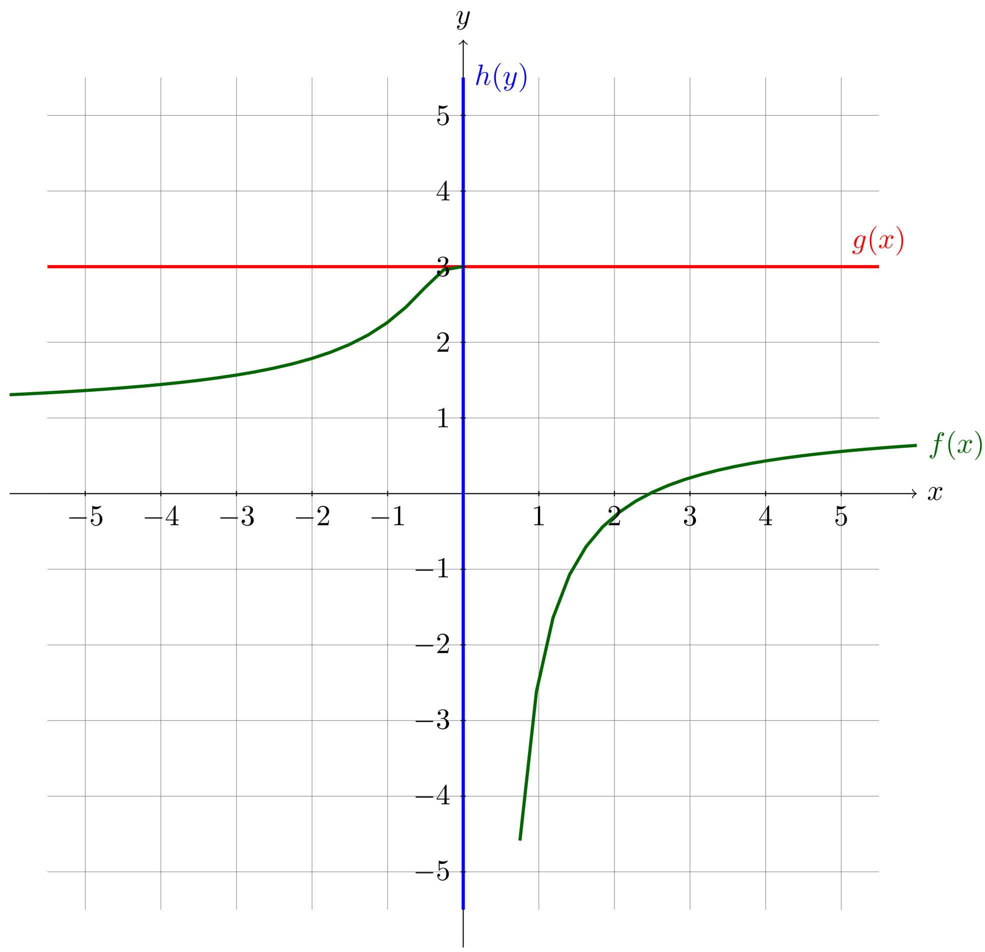


Solutions

General: $f(x) = Ce^{-\frac{1}{2x}}$ for $x \neq 0$ (here $C = 2$),

Particular: $g(x) = 0$ for $x \neq 0$,
$h(y) = 0$ for $y \neq 0$.

Equation: $x^2 \frac{\mathrm{d}y}{\mathrm{d}x} + y - a = 0$

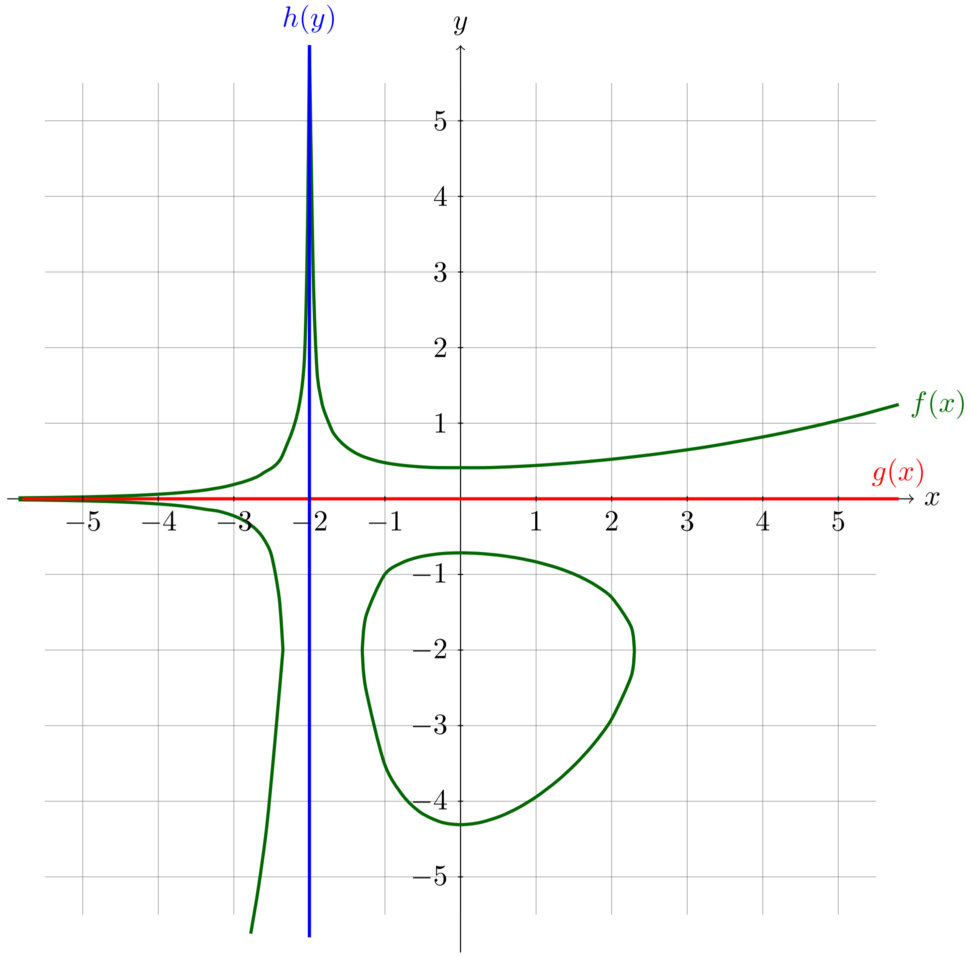


Solutions

General: $f(x) = a - Ce^{\frac{1}{x}}$, where $C \neq 0$ (here $a = 3$ and $C = 2$),

Particular: $g(x) = a$ for $x \neq 0$,
$h(y) = 0$ for $y \neq 0$.

Equation: $xy = (a+x)(b+y)\dfrac{\mathrm{d}y}{\mathrm{d}x}$

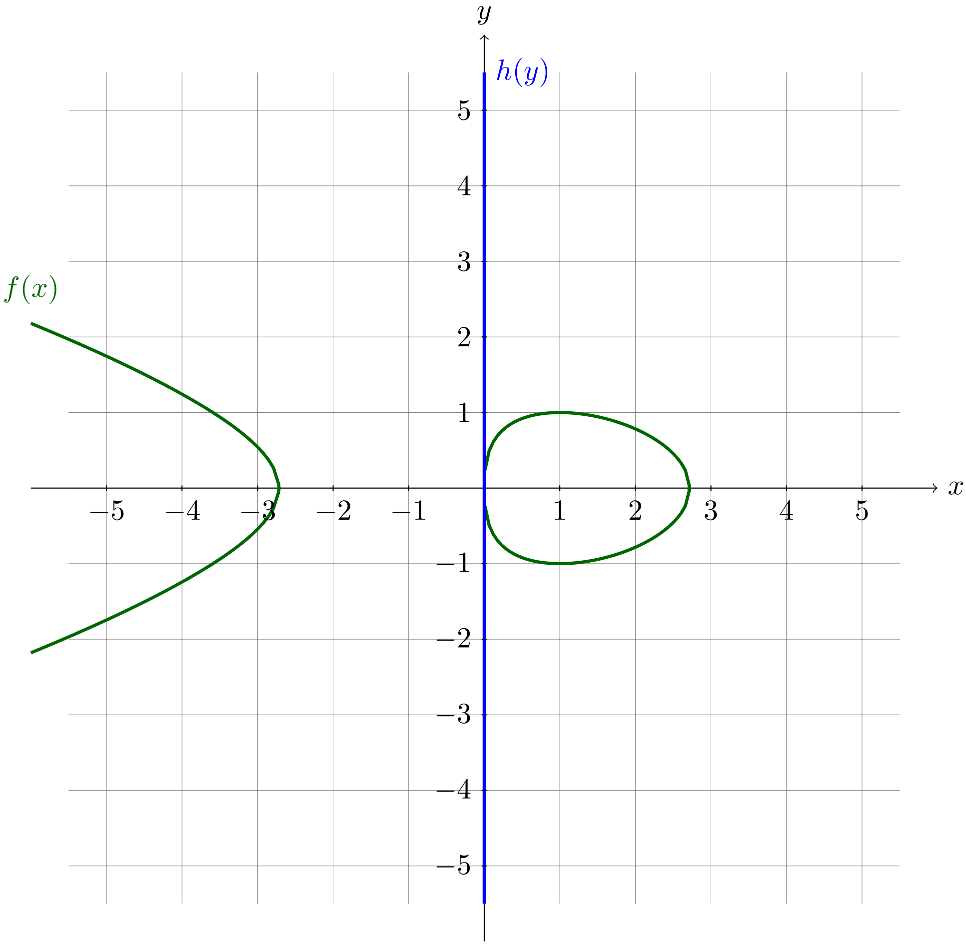


Solutions

General: $f(x)$ is such that $x - f(x) + C = \ln|f(x)|^b + \ln|x+a|^a$ (here $a = 2,\ b = 2,\ C = 0$).

Particular: $g(x) = 0$ for $x \neq -a$,
$h(y) = -a$ for $y \neq 0$.

Equation: $x - y^2 + 2xy\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

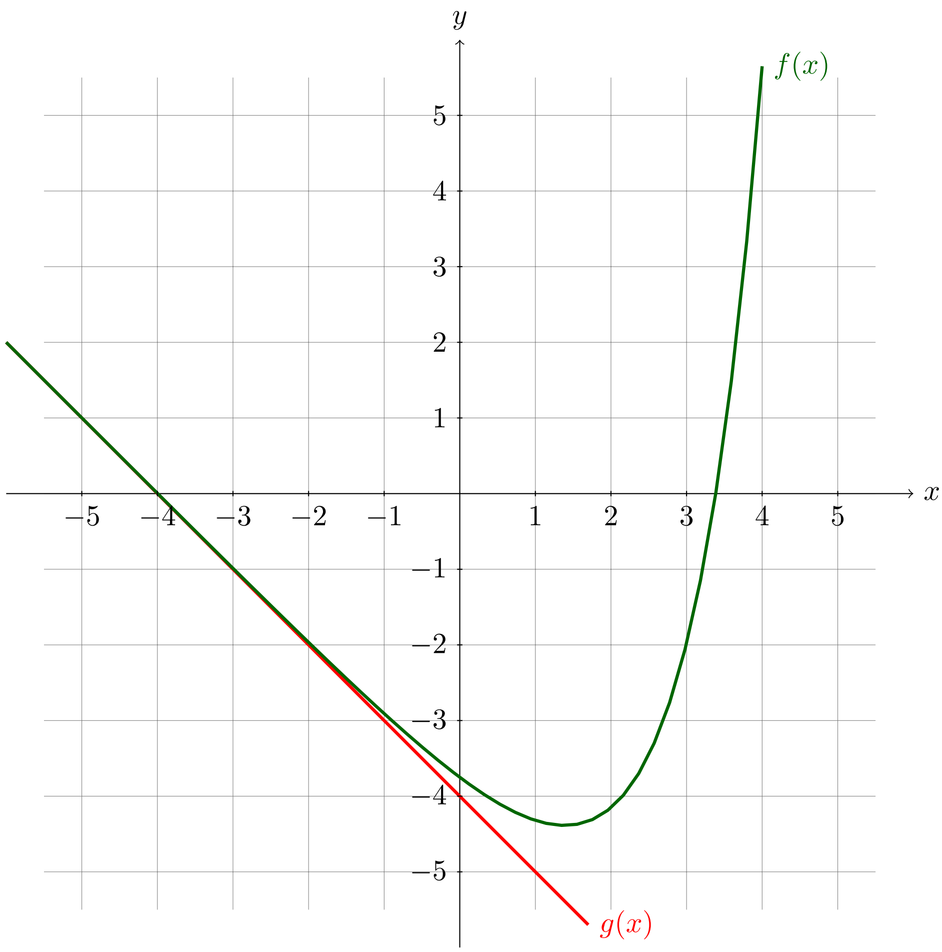


Solutions

General: $f(x)$ is such that $\frac{f(x)^2}{x} + \ln|x| = C$ for $x \neq 0$ (here $C = 1$),

Particular: $h(y) = 0$ for $y \neq 0$.

Equation: $\frac{\mathrm{d}y}{\mathrm{d}x} = x + y + 3$

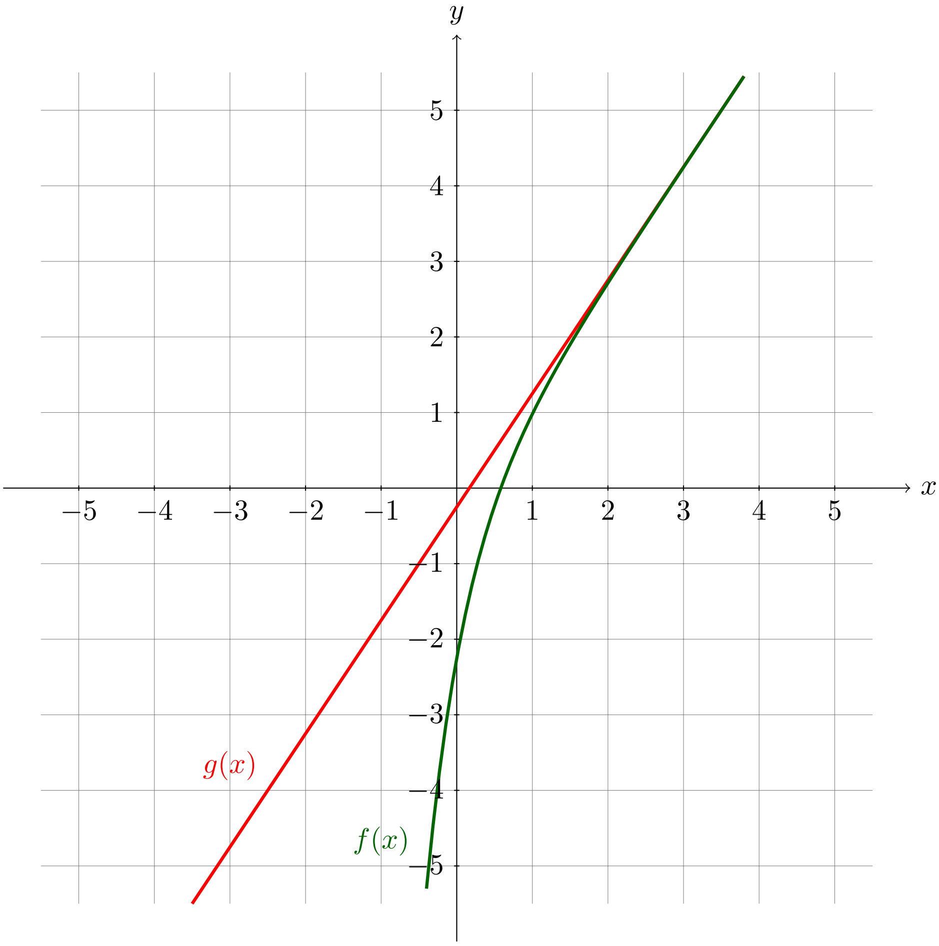


Solutions

General: $f(x) = Ce^x - x - 4$, where $C \neq 0$ (here $C = \frac{1}{4}$),

Particular: $g(x) = -x - 4$.

Equation: $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x - 2y + 1$

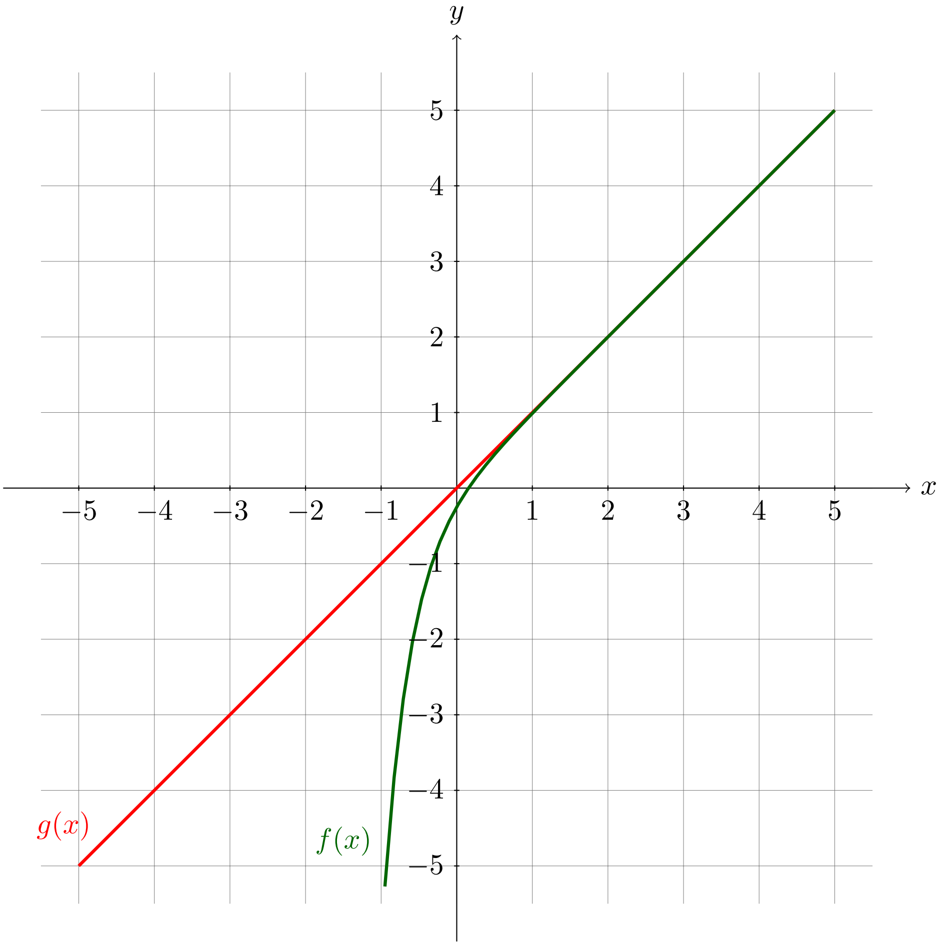


Solutions

General: $f(x) = Ce^{-2x} + \frac{3}{2}x - \frac{1}{4}$, where $C \neq 0$ (here $C = -2$),

Particular: $g(x) = \frac{3}{2}x - \frac{1}{4}$.

Equation: $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 5x - 3y + 7$

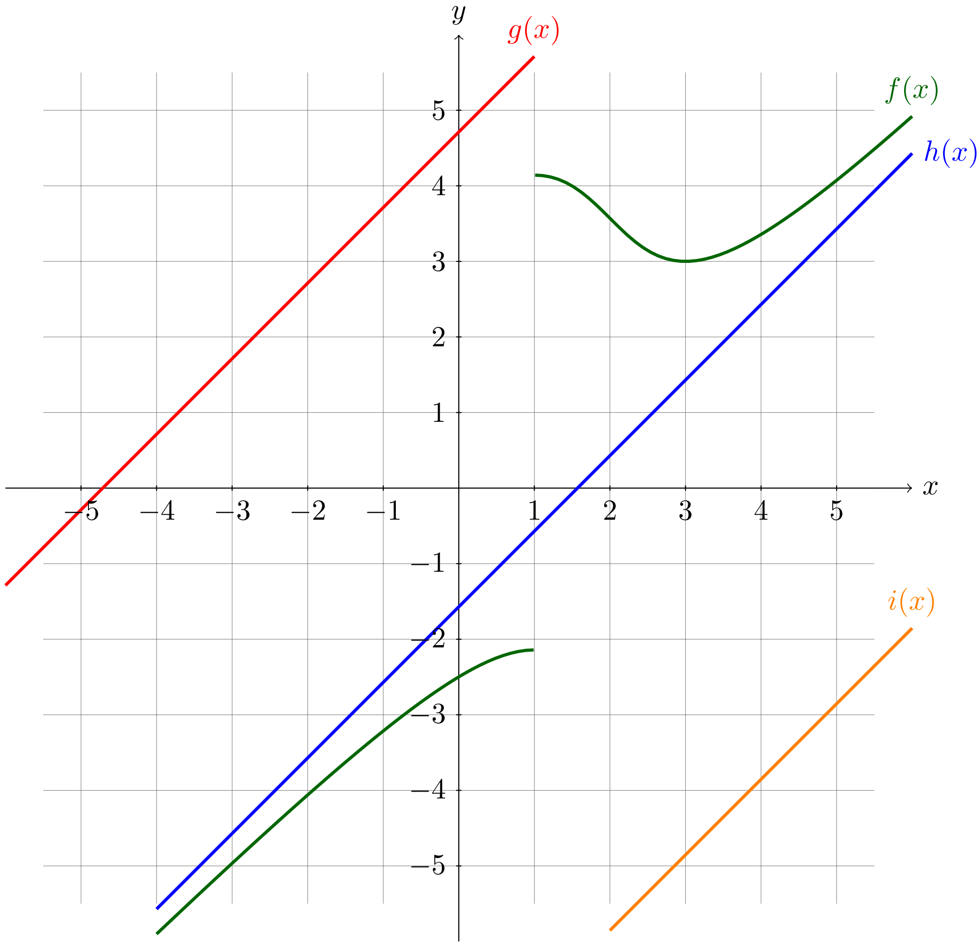


Solutions

General: $f(x) = Ce^{-3x} + \frac{5}{3}x - \frac{2}{9}$, where $C \neq 0$ (here $C = -\frac{1}{4}$),

Particular: $g(x) = \frac{5}{3}x - \frac{2}{9}$.

Equation: $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \sin(x-y)$

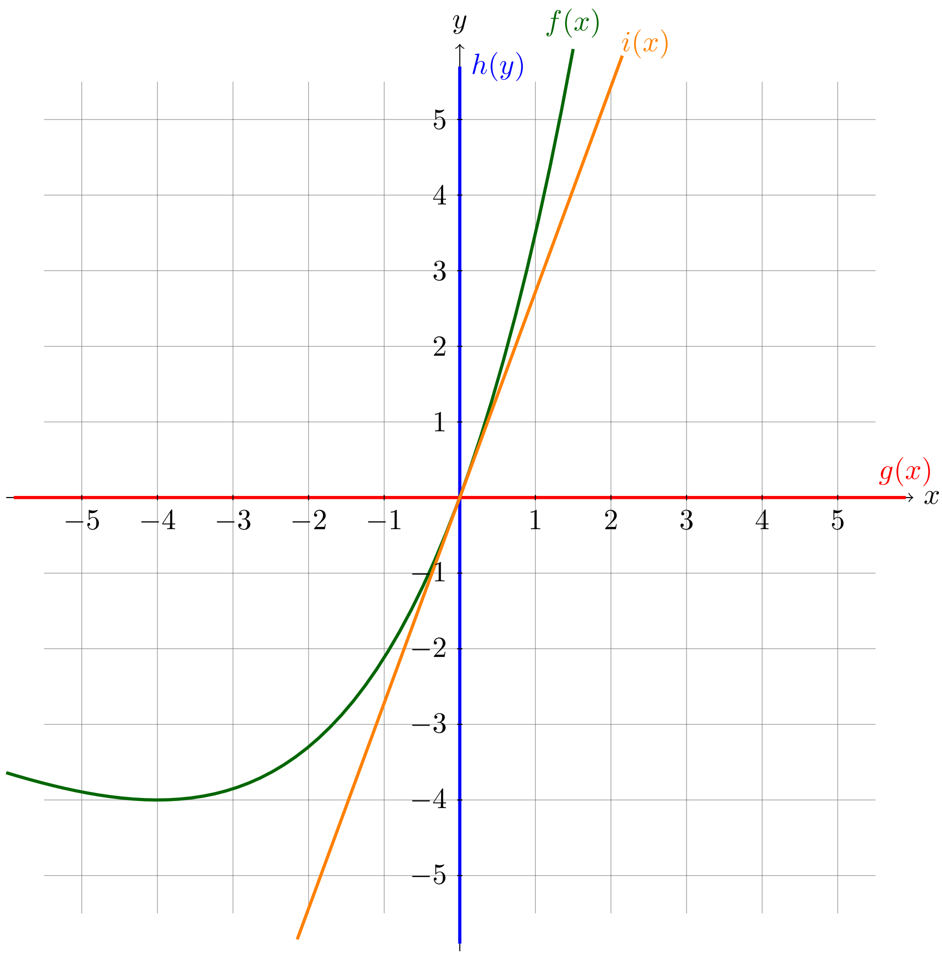


Solutions

General: $f(x) = x - 2\arctan\left(\frac{2}{C-x} + 1\right)$, where $x \neq C$ (here $C = 1$),

Particular is of the form $y = x - \frac{\pi}{2} + 2k\pi$, where $k \in \mathbb{Z}$, hence
$g(x) = x + \frac{3\pi}{2}$,
$h(x) = x - \frac{\pi}{2}$,
$i(x) = x - \frac{5\pi}{2}$.

Equation: $y' = \frac{y}{x} \ln \frac{y}{x}$

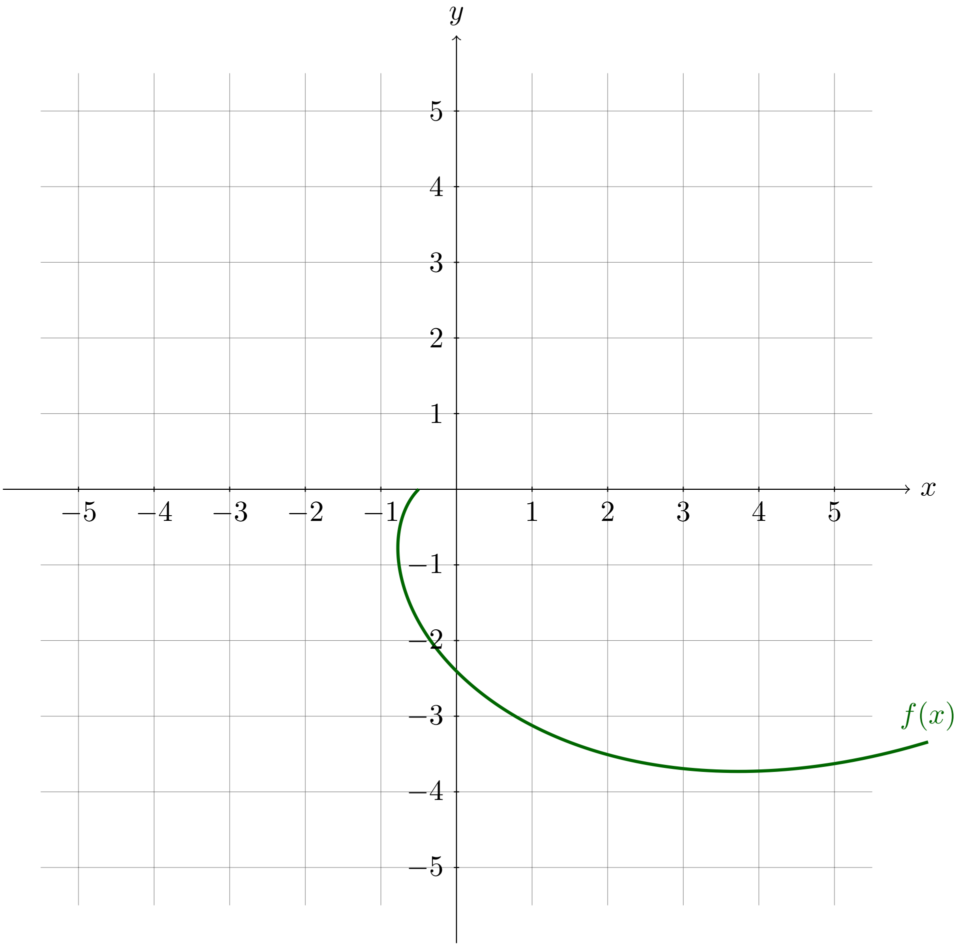


Solutions

General: $f(x) = xe^{Cx+1}$, where $C, x \neq 0$ (here $C = \frac{1}{4}$),

Particular: $g(x) = 0$ for $x \neq 0$,
$h(y) = 0$ for $y \neq 0$,
$i(x) = ex$ for $x \neq 0$.

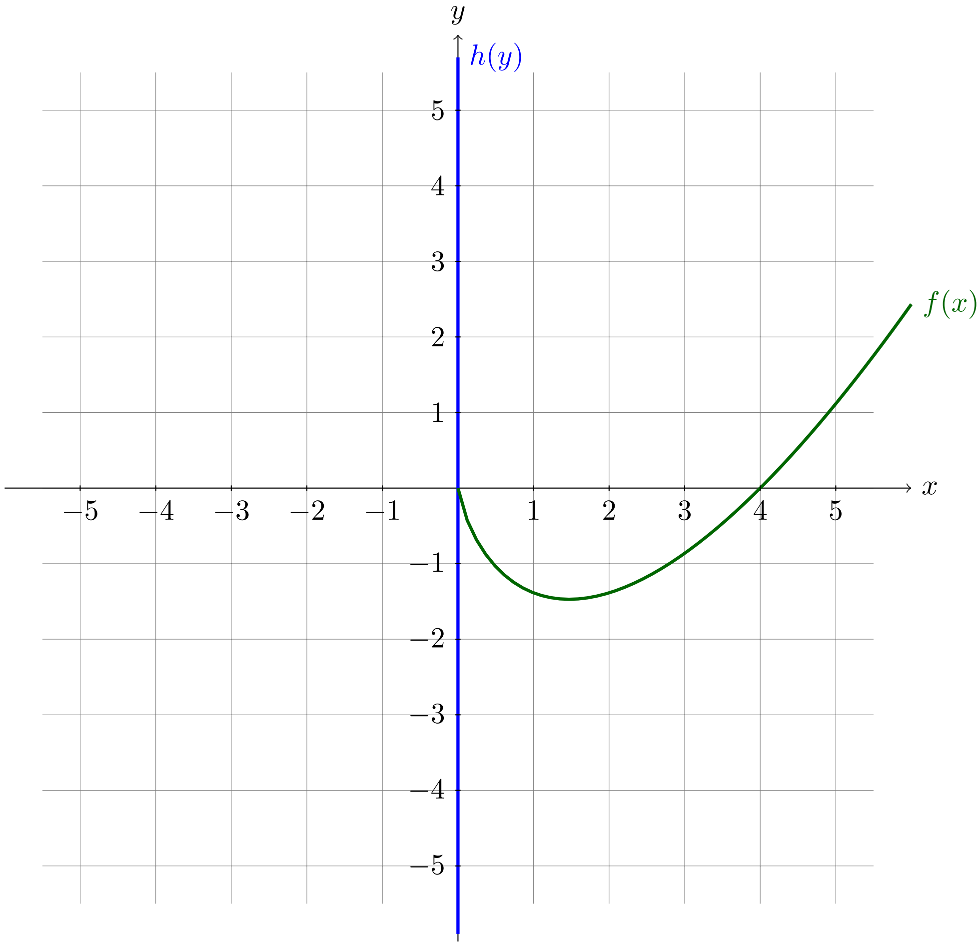
Equation: $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{x+y}{x-y}$



Solutions

General: $f(x)$ is such that $\sqrt{x^2 + f(x)^2} = Ce^{\arctan \frac{f(x)}{x}}$, where $C > 0$, or in polar coordinates $r = De^{\varphi}$ (here $D = -\frac{1}{2}$)

Equation: $x\dfrac{\mathrm{d}y}{\mathrm{d}x} = x + y$

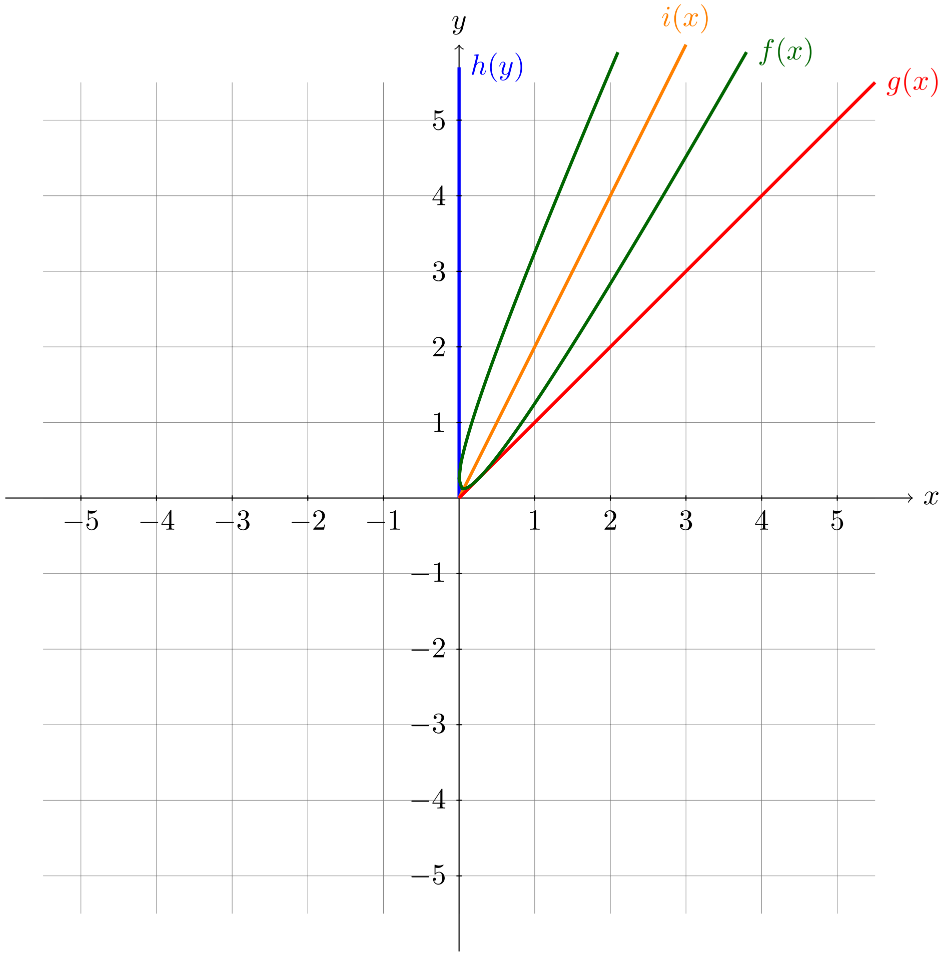


Solutions

General: $f(x) = x\ln(Cx)$, where $C \neq 0$ (here $C = \frac{1}{4}$),

Particular: $h(y) = 0$ for $y \neq 0$.

Equation: $y'\sqrt{x} = \sqrt{y-x} + \sqrt{x}$

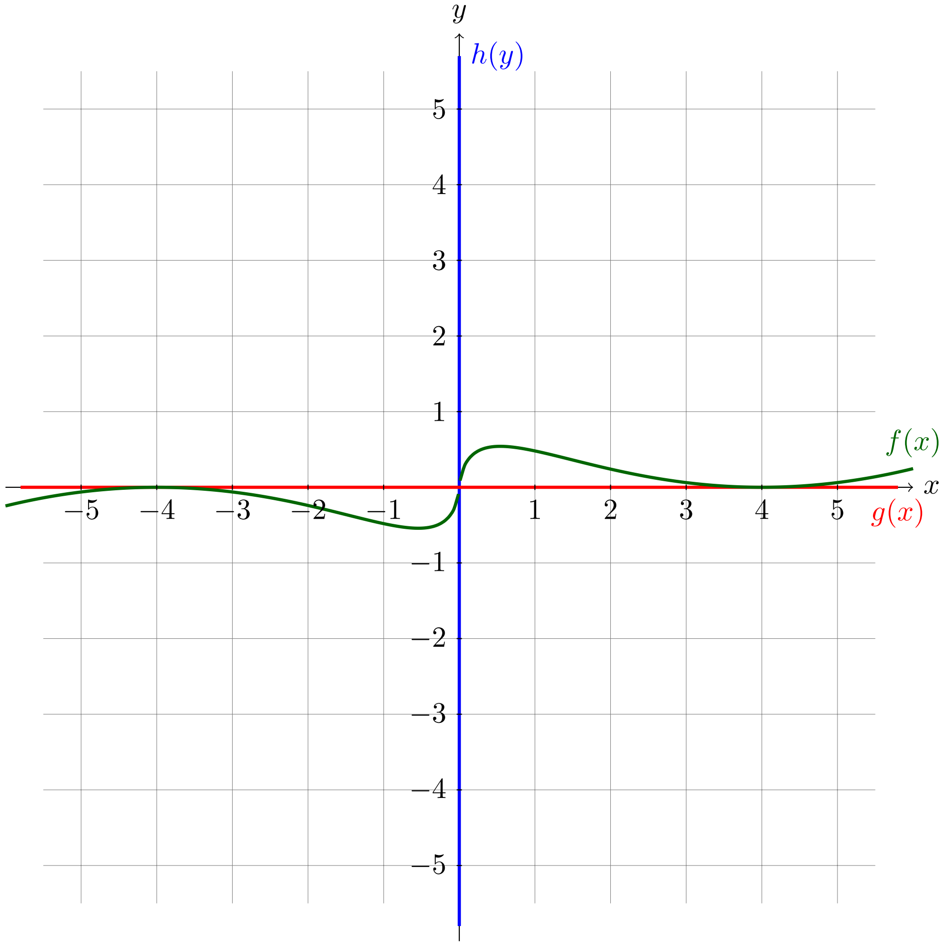


Solutions

General: $f(x)$ is such that $\left(\sqrt{x} - \sqrt{f(x)-x}\right)^2 = C$, where $C > 0$, (here $C = \frac{1}{2}$),

Particular: $g(x) = x$ for $x > 0$ (singular solution),
$h(y) = 0$ for $y > 0$,
$i(x) = 2x$ for $x > 0$.

Equation: $(y + \sqrt{xy})\mathrm{d}x = x\mathrm{d}y$,

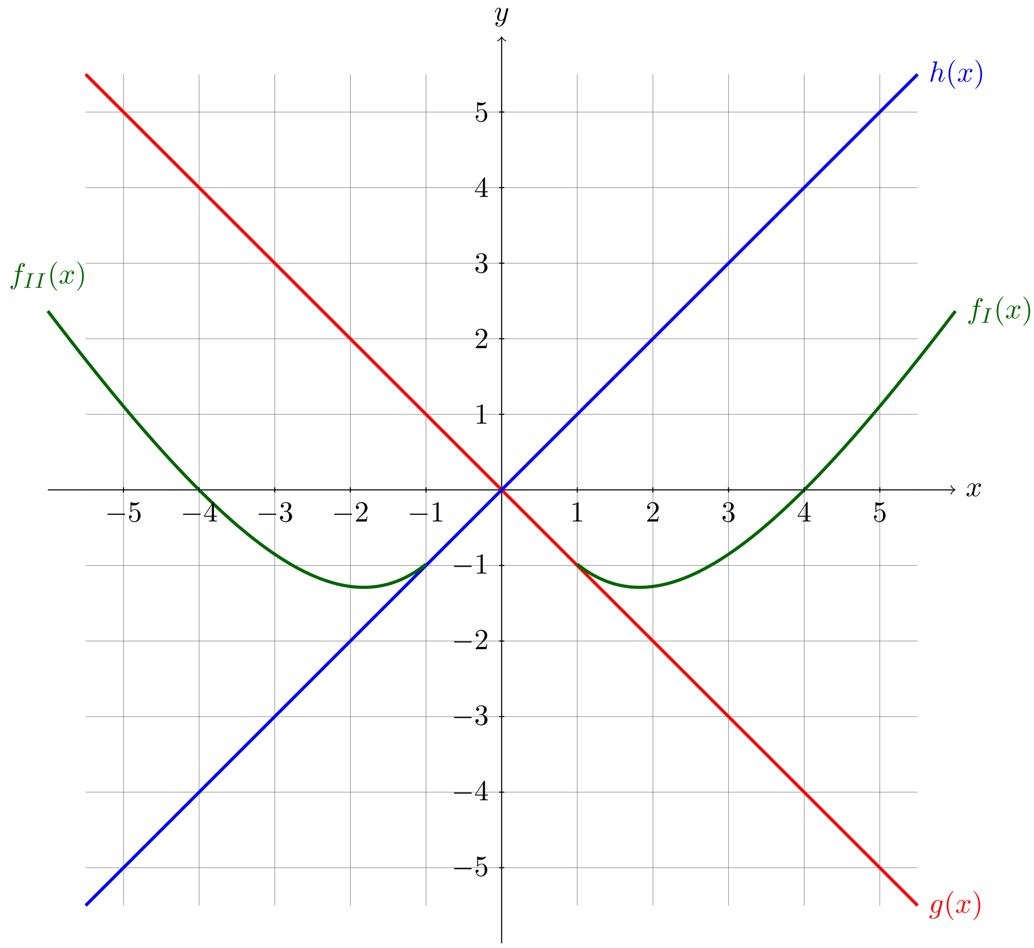


Solutions

General: $f(x) = \frac{1}{4}x \ln(C\,|x|)$, where $C > 0$ and $x \neq 0$, (here $C = \frac{1}{4}$),

Particular: $g(x) = 0$ for $x \neq 0$ (singular solution),
$h(y) = 0$ for $y \neq 0$.

$xy' = \sqrt{x^2 - y^2} + y$

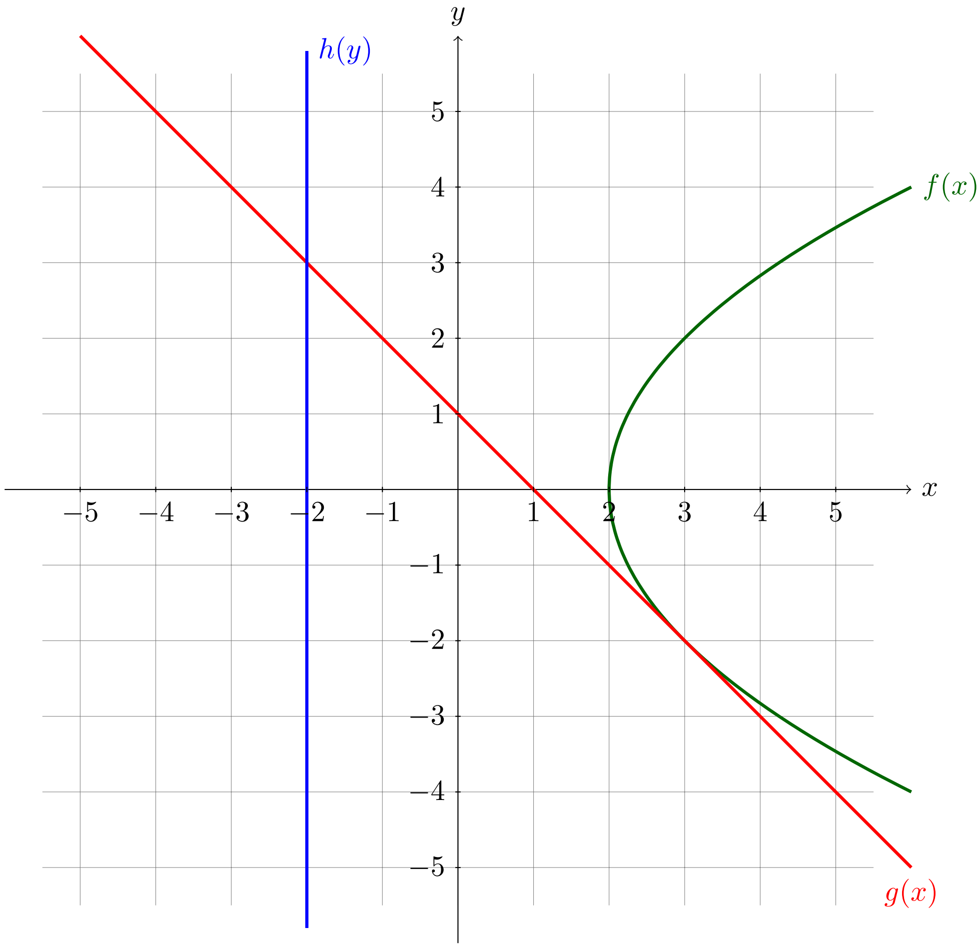


Solutions

General: $f_I(x) = x\sin(\ln Cx)$, where $\frac{1}{C}e^{-\frac{\pi}{2}} \le x \le \frac{1}{C}e^{\frac{\pi}{2}}$ and $C > 0$, (here $C = \frac{1}{4}$),
$f_{II}(x) = x\sin(-\ln Dx)$, where $\frac{1}{D}e^{\frac{\pi}{2}} \le x \le \frac{1}{D}e^{-\frac{\pi}{2}}$ and $D < 0$, (here $D = -\frac{1}{4}$),

Particular: $g(x) = -x$ for $x \neq 0$ (singular solution),
$h(x) = x$ for $x \neq 0$ (singular solution).

Equation: $(y+2)\mathrm{d}x = (2x+y-4)\mathrm{d}y$

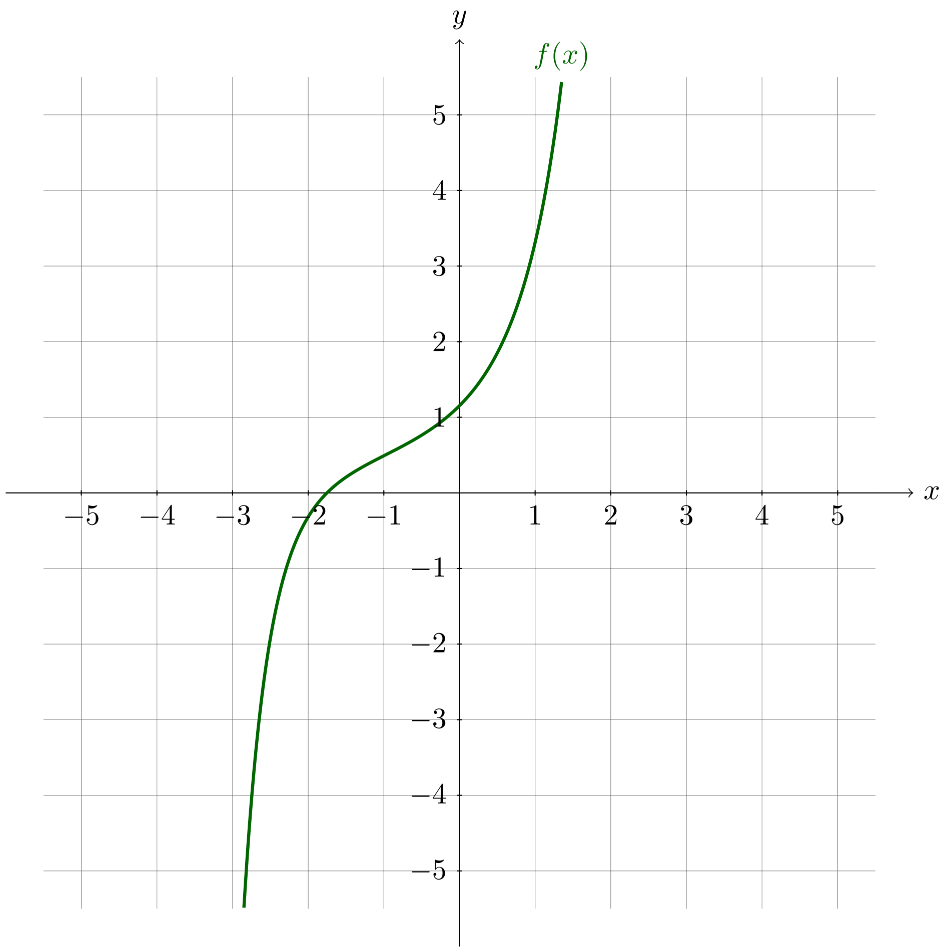


Solutions

General: $f(x)$ is such that $C(f(x)+2)^2 = f(x)+x-1$ (here $C=\frac{1}{4}$),

Particular: $g(x) = 1-x$ for $x \neq 3$,
$h(y) = -2$.

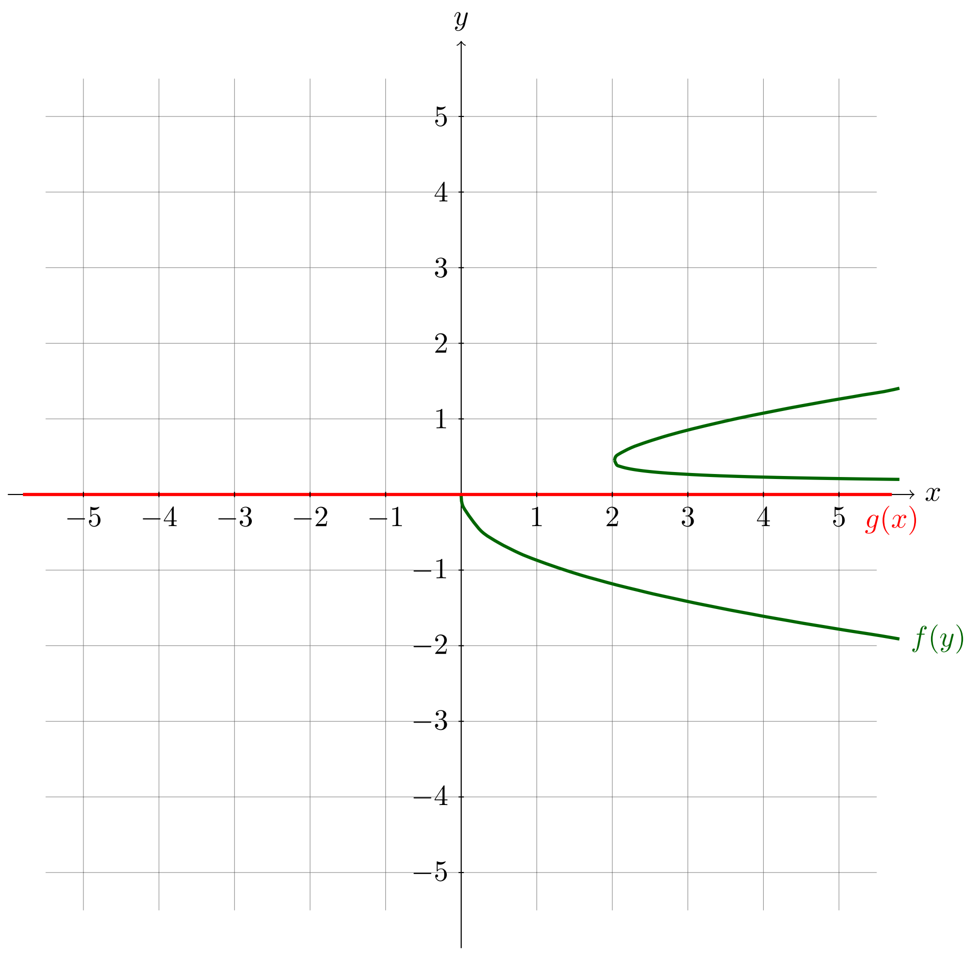
Equation: $y' - xy = 1$



Solutions

General: $f(x) = \left(C + \int e^{-\frac{1}{2}x^2}\mathrm{d}x\right) e^{\frac{1}{2}x^2}$, (here $C = -\frac{1}{10}$).

Equation: $(x - 2xy - y^2)\mathrm{d}y + y^2\mathrm{d}x = 0$

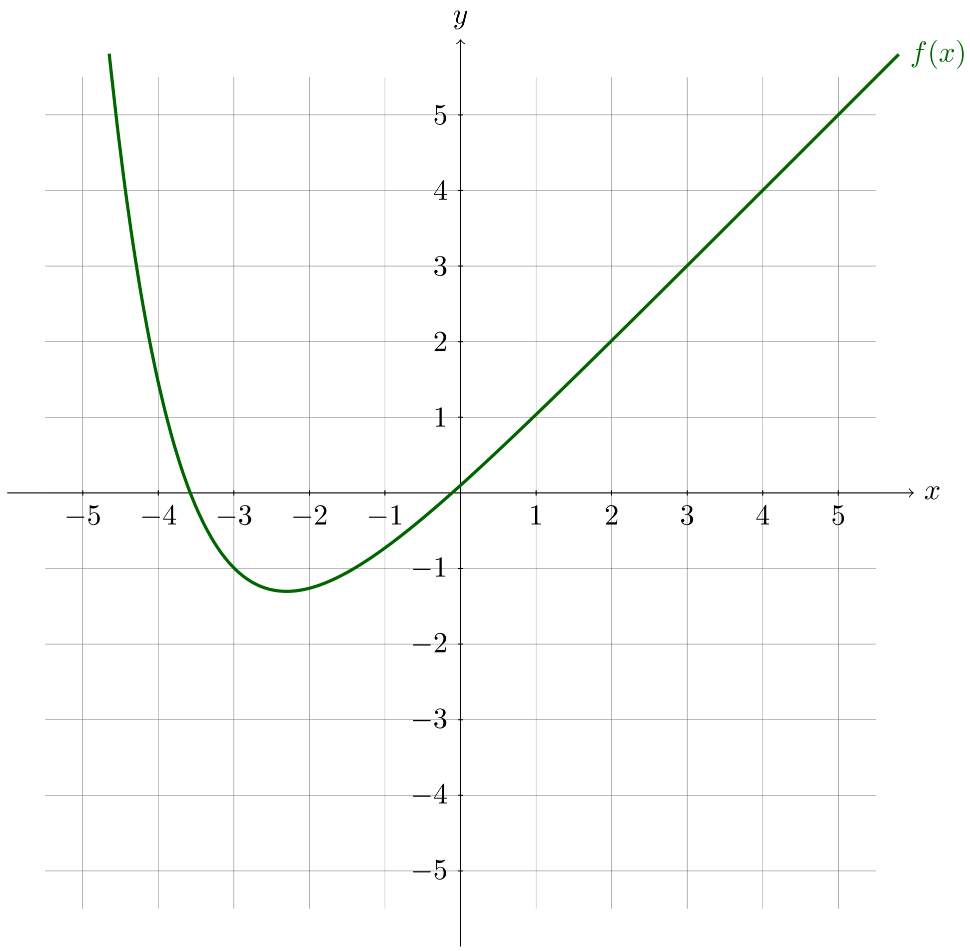


Solutions

General: $f(y) = y^2 + Cy^2 e^{\frac{1}{y}}$ (here $C = 1$),

Particular: $g(x) = 0$ for $x \neq 0$.

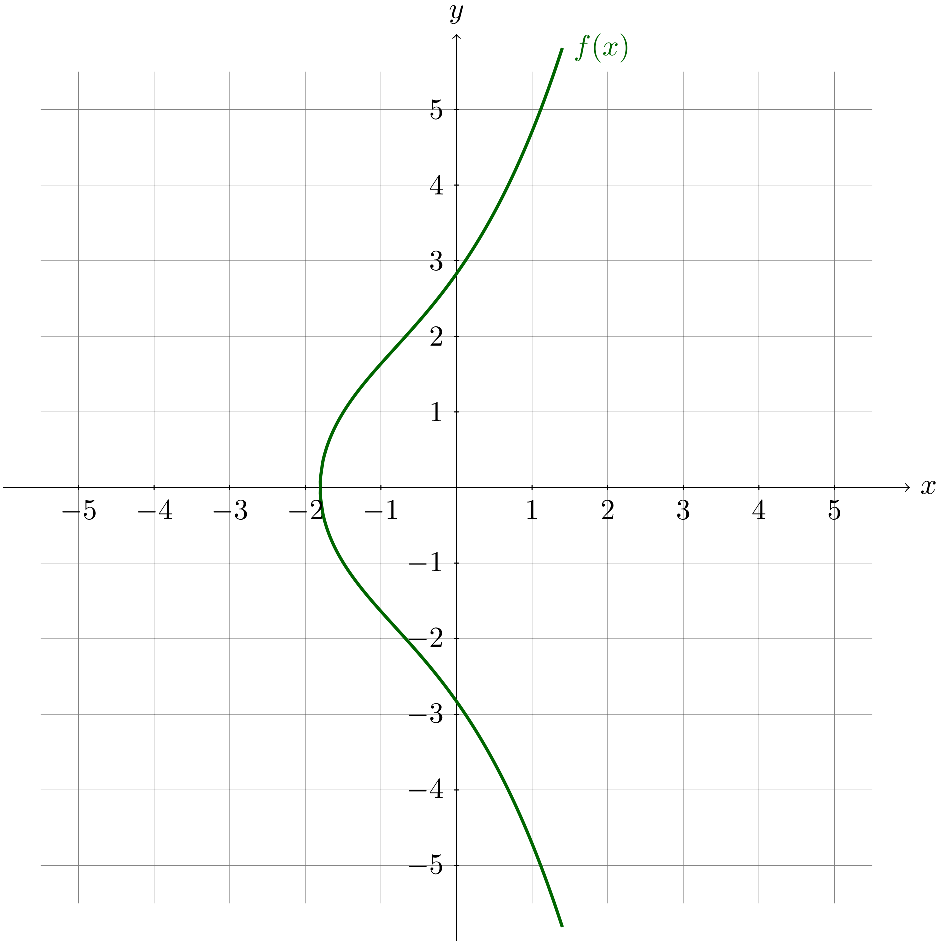
Equation: $y' + y = x + 1$



Solutions

General: $f(x) = x + Ce^{-x}$ (here $C = \frac{1}{10}$).

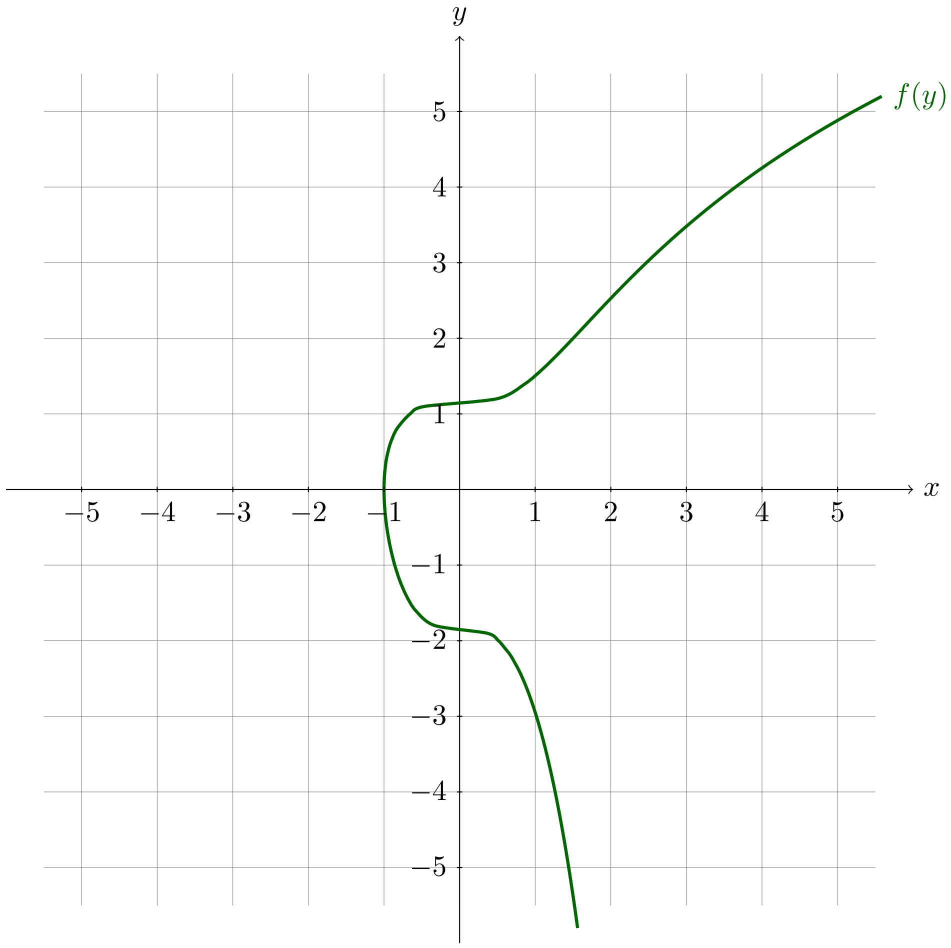
Equation: $yy' = \frac{1}{2}(x^2 + y^2)$



Solutions

General: $f(x)$ is such that $f(x)^2 = Ce^x - x^2 - 2x - 2$ (here $C = 10$).

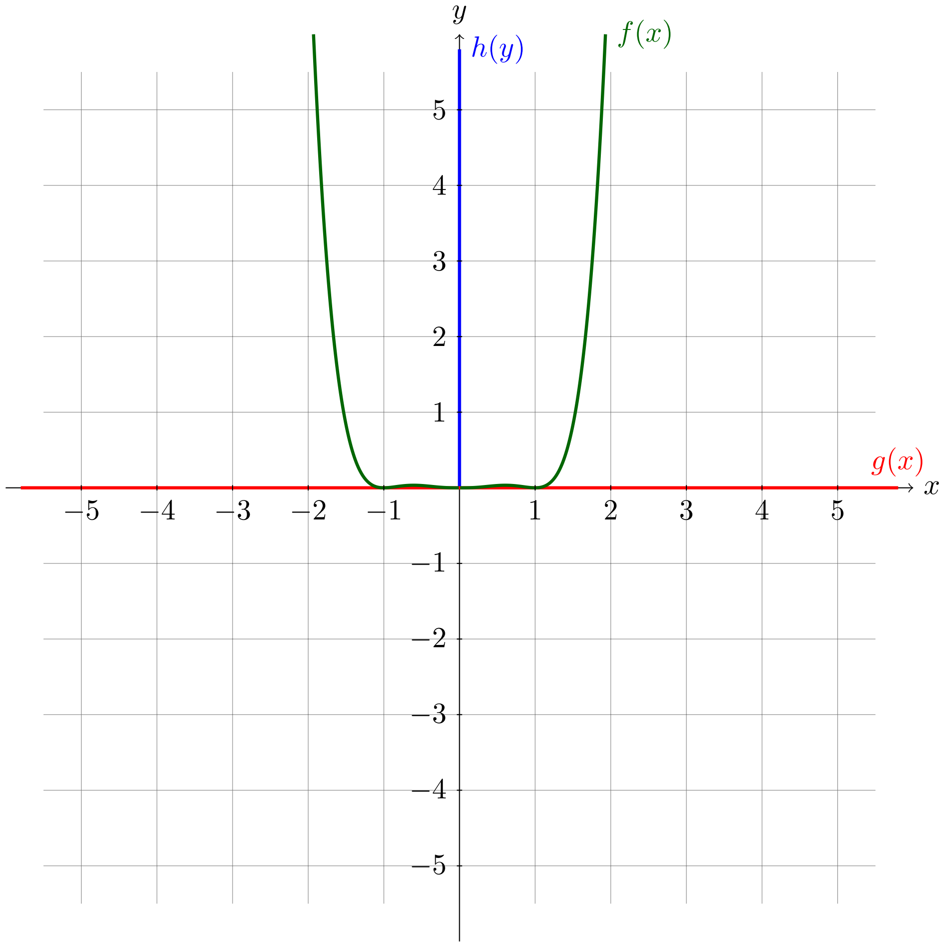
Equation: $(x^3 + y + 1)\mathrm{d}y - 3x^2\mathrm{d}x = 0$



Solutions

General: $f(y) = \sqrt[3]{-y - 2 + Ce^{y}}$, where $C \neq 0$, (here $C = 1$).

Equation: $\dfrac{xy'}{\sqrt{y}} = 4\sqrt{y} + 2x^2$

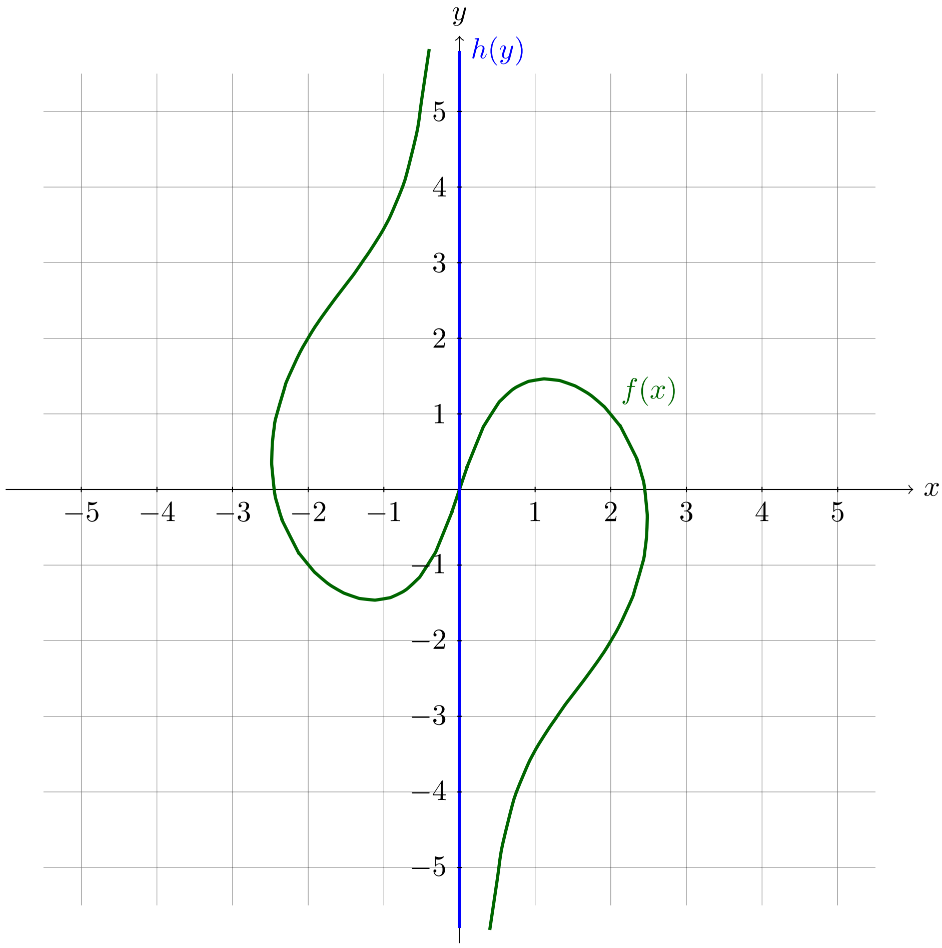


Solutions

General: $f(x)$ is such that $\sqrt{f(x)} = x^2\left(\ln|x| + C\right)$, where $x \neq 0$, (here $C = 0$),

Particular: $g(x) = 0$ for $x \neq 0$ (singular solution),
$h(y) = 0$ for $y > 0$.

Equation: $(x^3 - y)\mathrm{d}x + (x^2 y + x)\mathrm{d}y = 0$

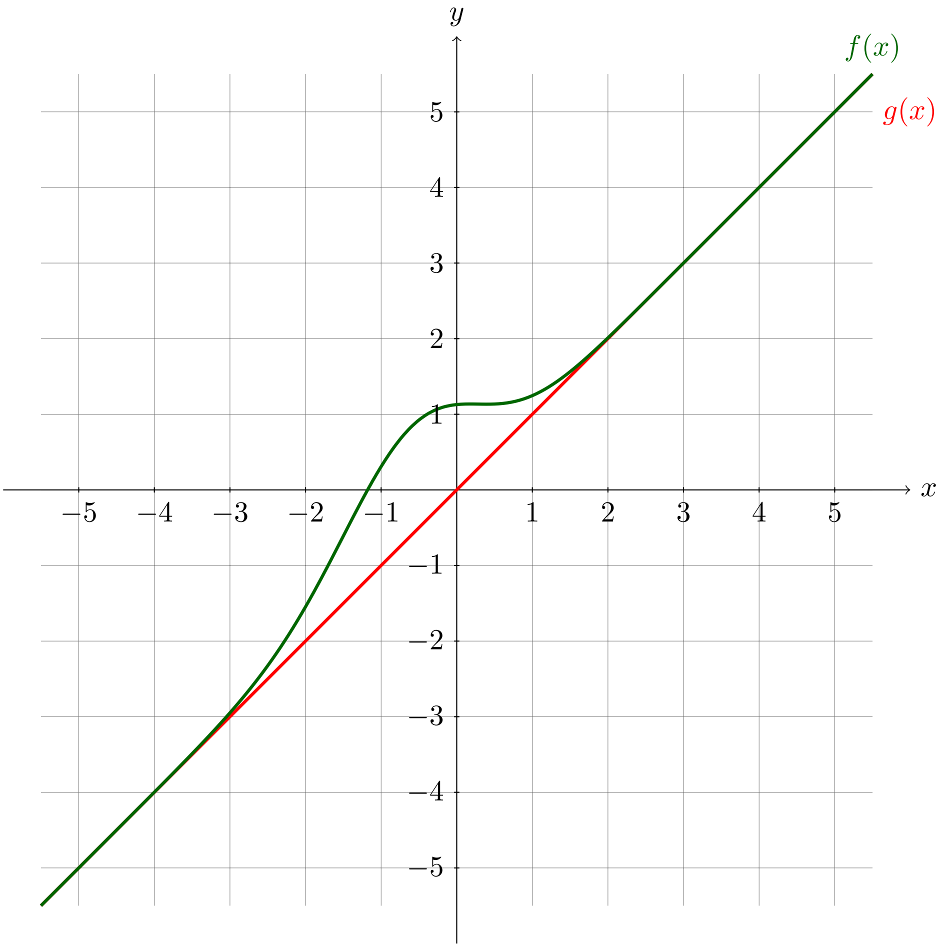


Solutions

General: $f(x)$ is such that $x(f(x)^2 + x^2) + 2f(x) = Cx$, where $C \neq 0$, (here $C = 6$),

Particular: $h(y) = 0$ for $y \neq 0$.

Equation: $y' = -y^2 + 1 + x^2$

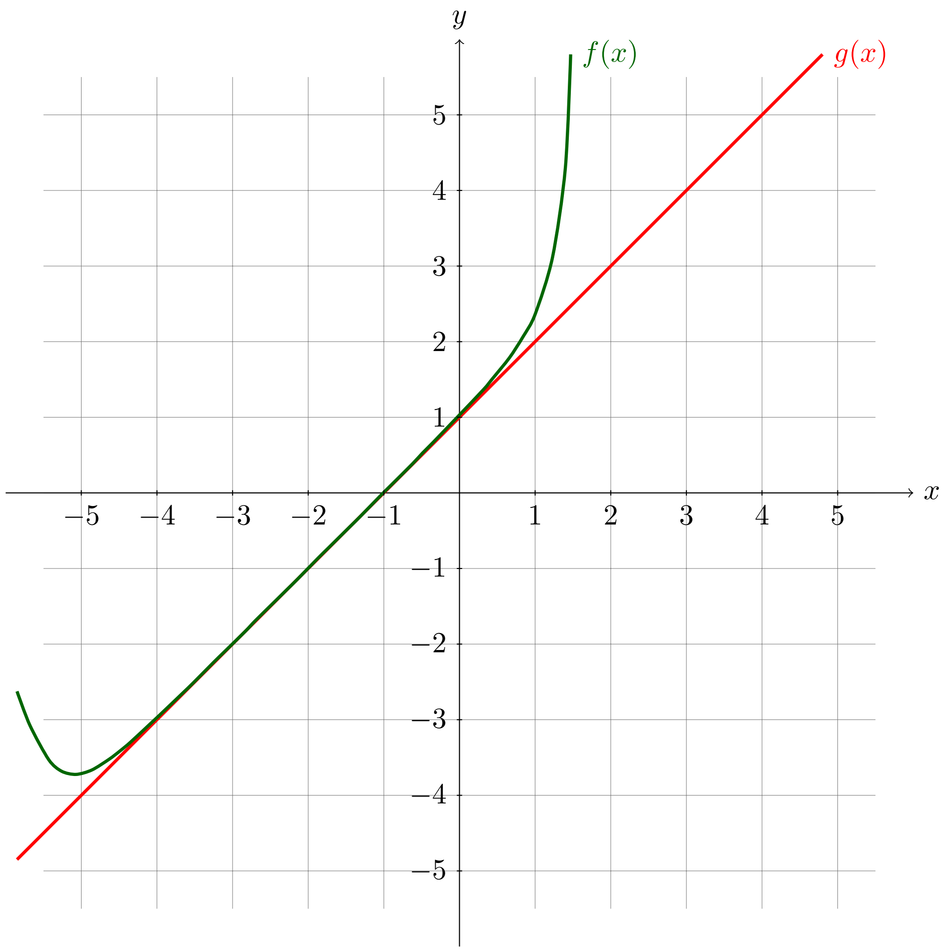


Solutions

General: $f(x) = x + \frac{e^{-x^2}}{C + \int e^{x^2} \mathrm{d}x}$ (here $C = 1$),

Particular: $g(x) = x$.

Equation: $y' = y^2 - xy - x$

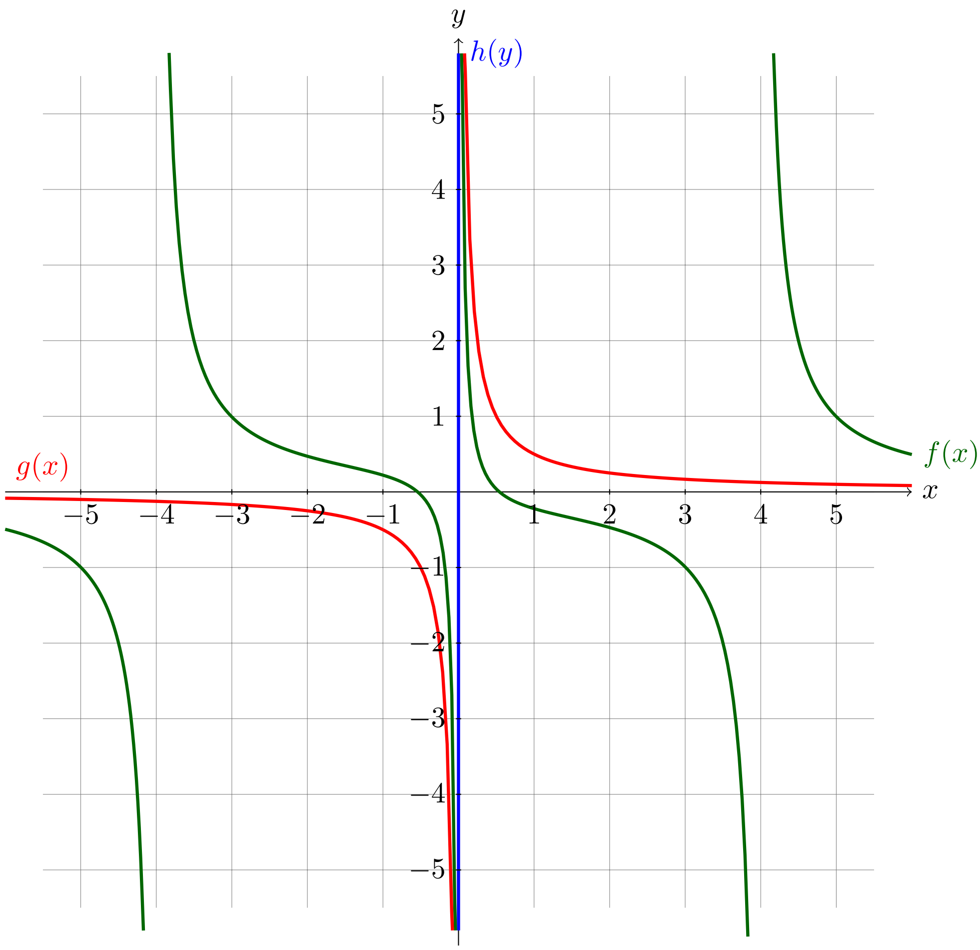


Solutions

General: $f(x) = x + 1 + \left(e^{\frac{1}{2}x^2+2x}\right)\left(-\int e^{\frac{1}{2}x^2+2x}\mathrm{d}x + C\right)^{-1}$
(here $f(x)$ is such that $f(0) = 1$),

Particular: $g(x) = x + 1.$

Equation: $y' + y^2 = -\frac{1}{4x^2}$

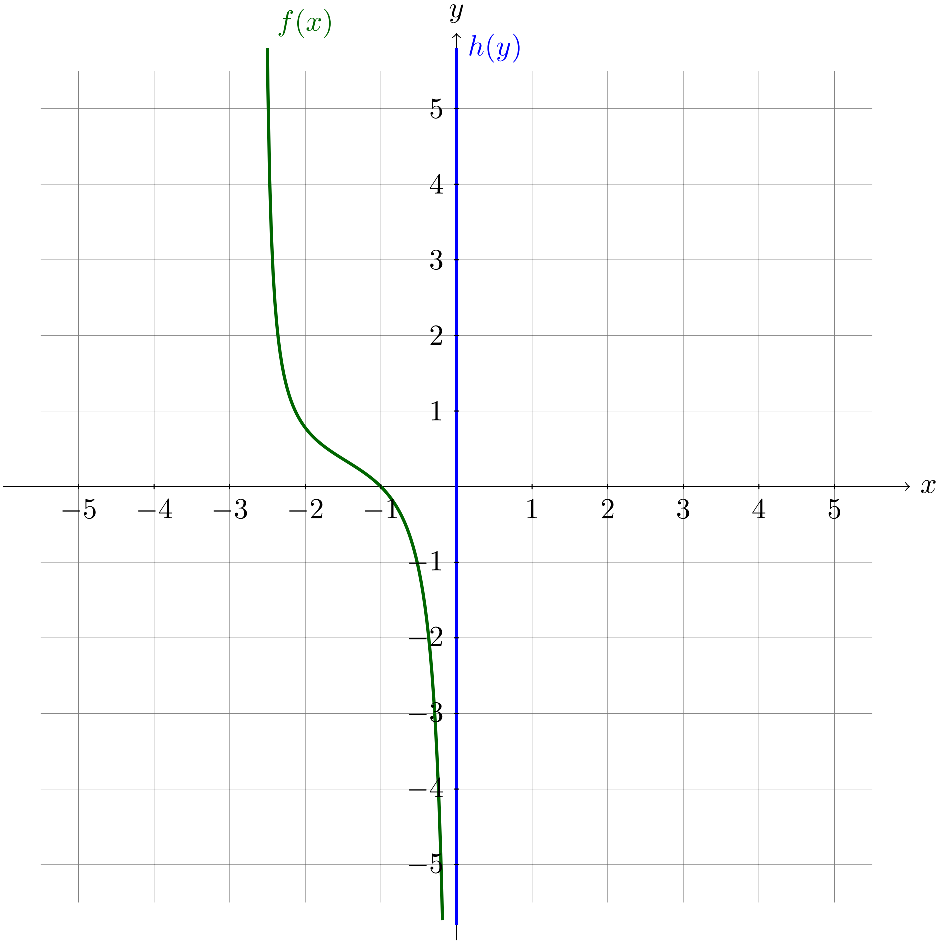


Solutions

General: $f(x) = \frac{1}{x \ln C|x|} + \frac{1}{2x}$ (here $C = \frac{1}{4}$),

Particular: $g(x) = \frac{1}{2x}$ for $x \neq 0$,
$h(y) = 0$ for $y \neq 0$.

Equation: $xy' = x^2y^2 - y + 1$

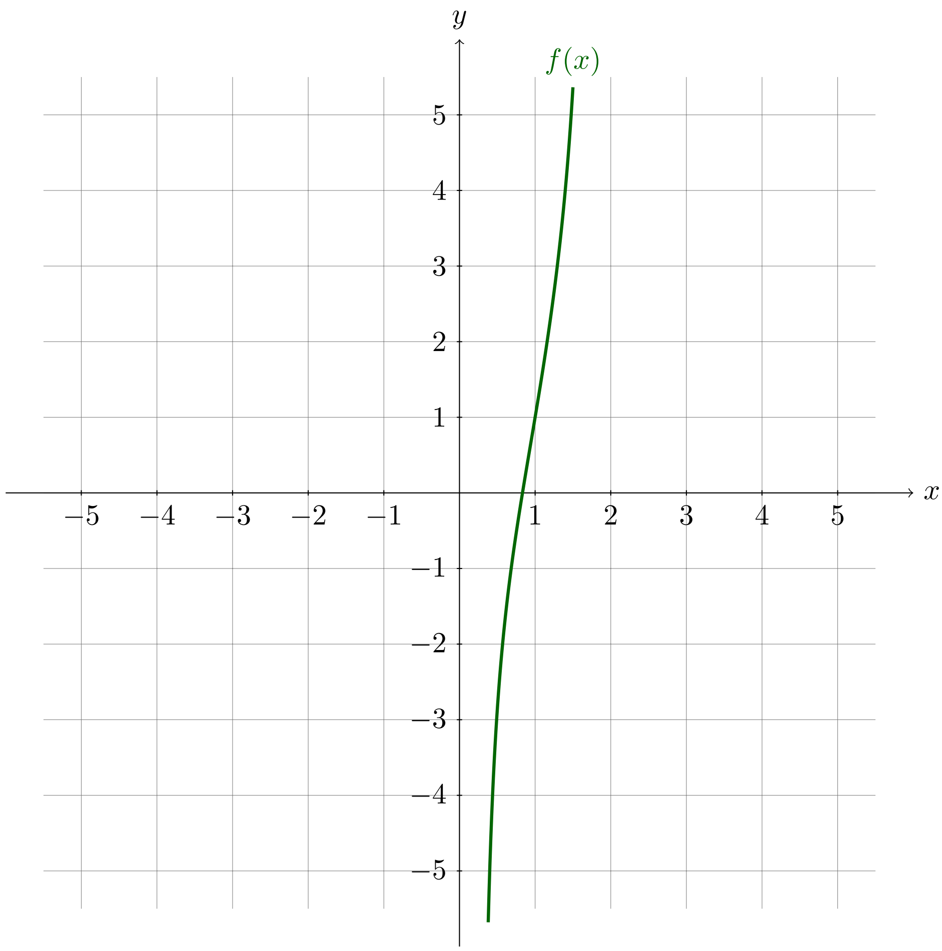


Solutions

General: $f(x) = \frac{1}{x}\tan(x + C)$, where $-C - \frac{\pi}{2} < x < -C + \frac{\pi}{2}$, (here $C = 1$),

Particular: $h(y) = 0$ for $y \neq 1$.

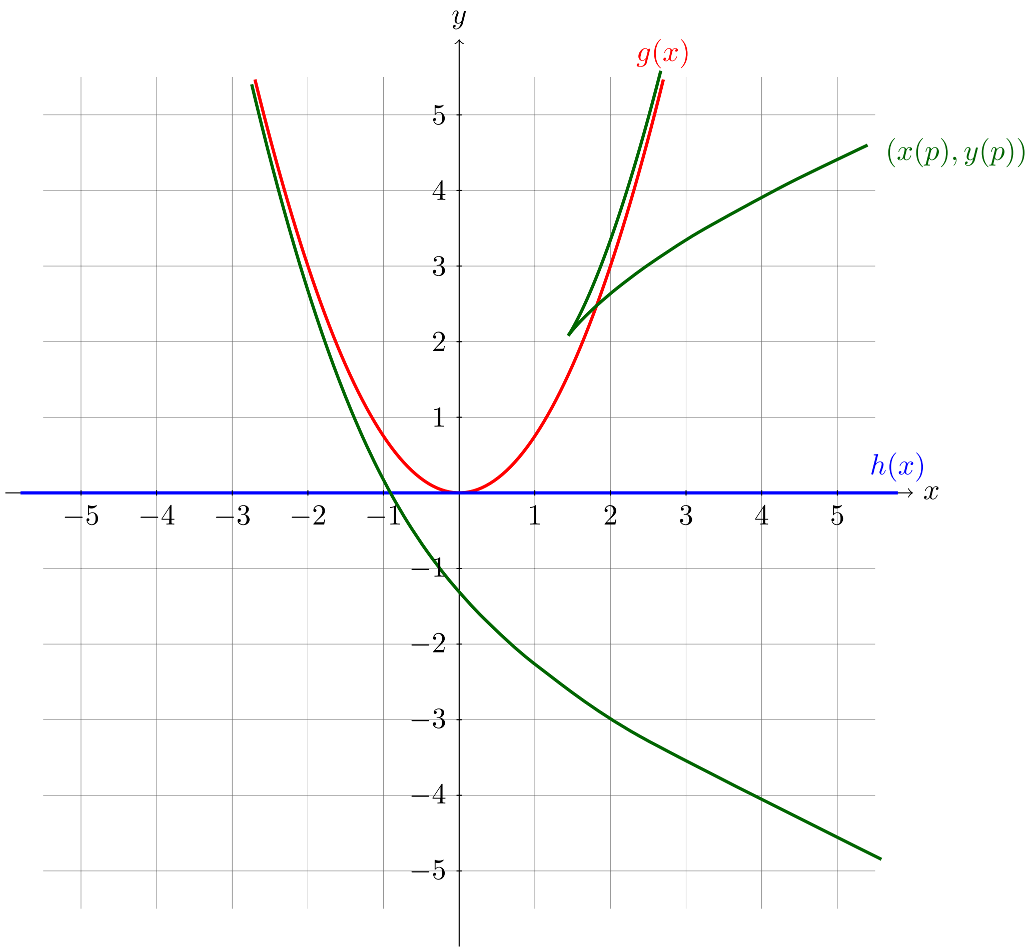
Equation: $y' = y^2 - 2x^2y + x^4 + 2x + 4$



Solutions

General: $f(x) = x^2 + 2\tan(2x + C)$, where $-\frac{C}{2} - \frac{\pi}{4} < x < -\frac{C}{2} + \frac{\pi}{4}$, (here $C = -2$).

Equation: $y = 2xy' - y'^2$

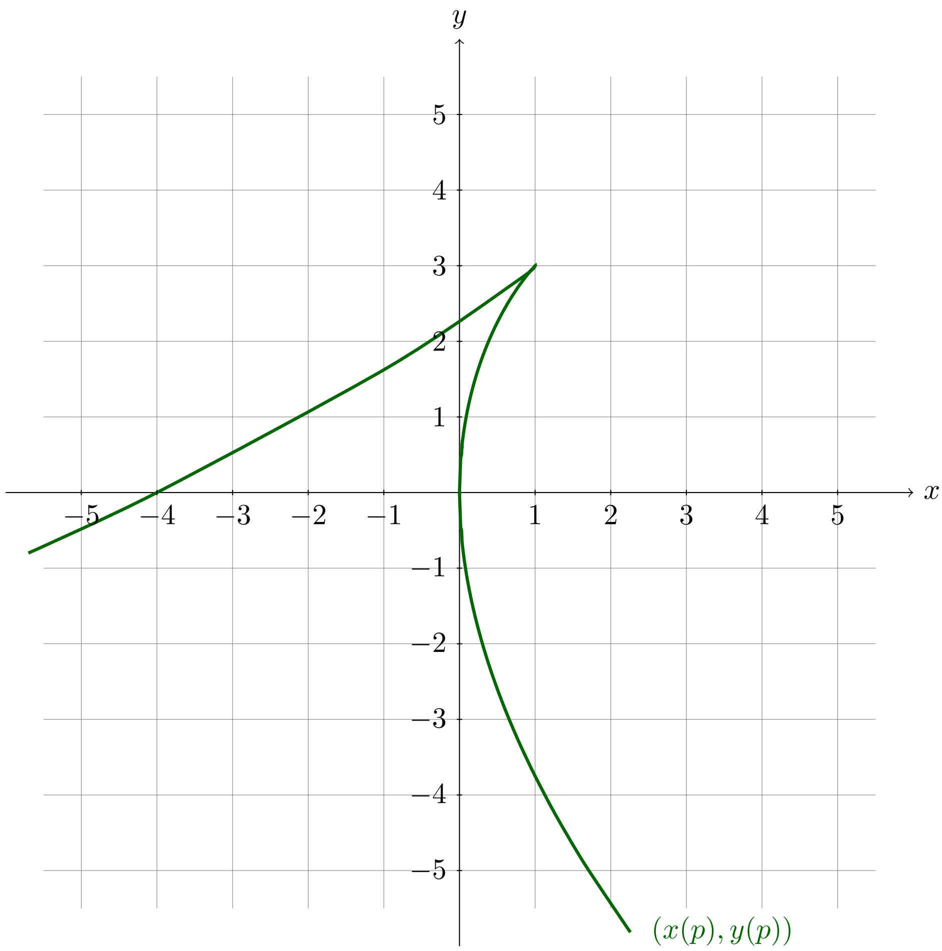


Solutions

General: $x(p) = \frac{2}{3}p + \frac{C}{p^2}$ and $y(p) = \frac{p^2}{3} + \frac{2C}{p}$ (here $C = 1$),

Particular: $g(x) = \frac{3}{4}x^2$,
$h(x) = 0$.

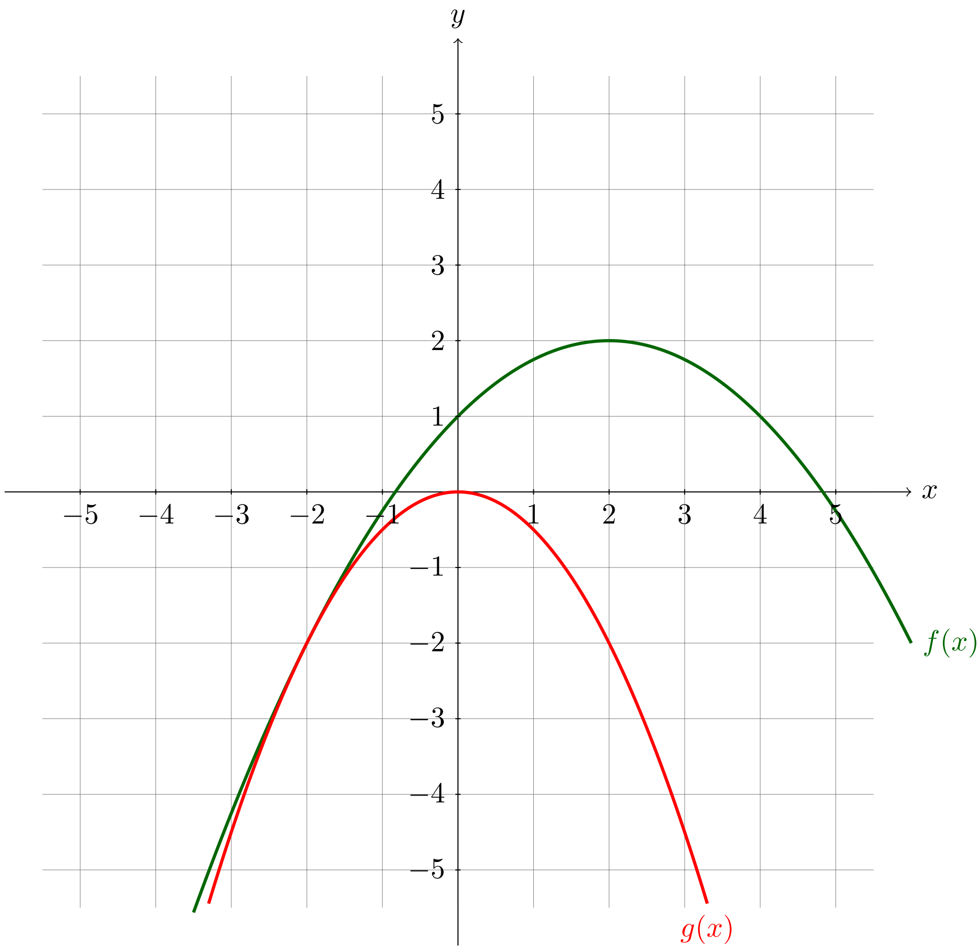
Equation: $y = 2xy' + \dfrac{1}{y'^2}$



Solutions

General: $x(p) = -\frac{2}{p^3} + \frac{C}{p^2}$ and $y(p) = -\frac{3}{p^2} + \frac{2C}{p}$ (here $C = 3$).

Equation: $y = 2xy' + \dfrac{1}{2}x^2 + y'^2$

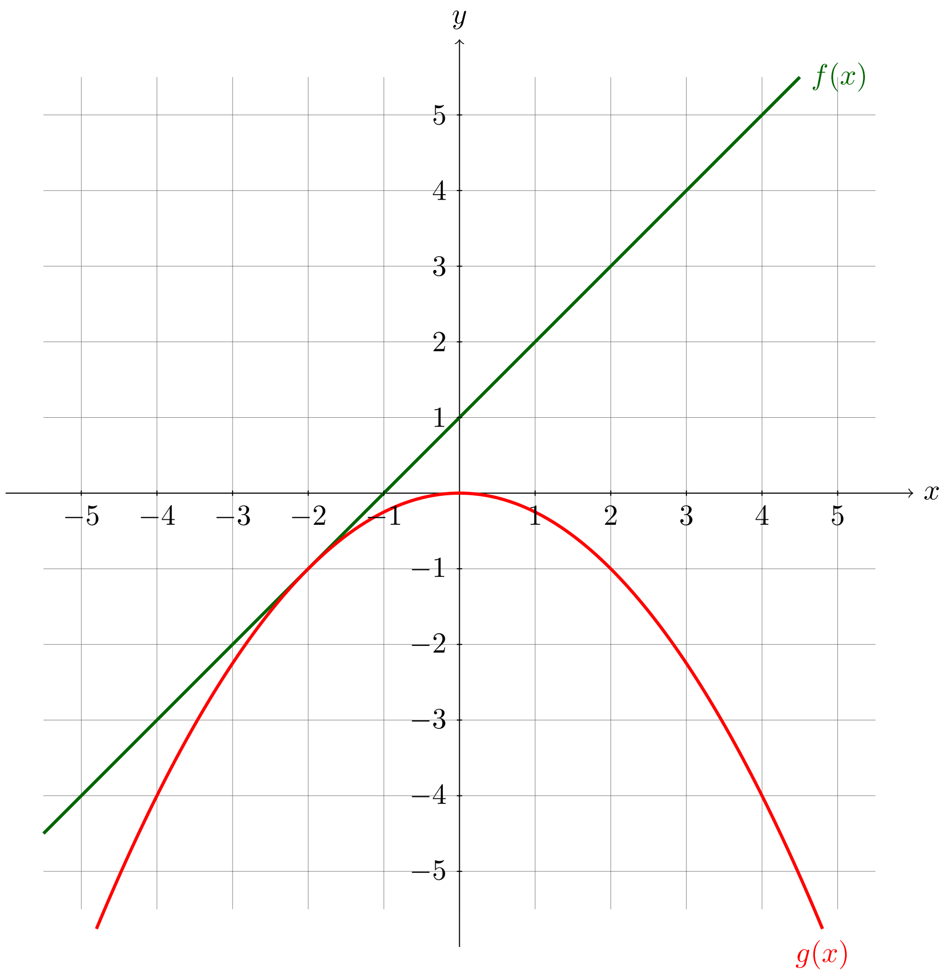


Solutions

General: $f(x) = -\frac{1}{4}x^2 + Cx + C^2$ (here $C = 1$),

Particular: $g(x) = -\frac{1}{2}x^2$ (singular solution).

Equation: $y = xy' + y'^2$

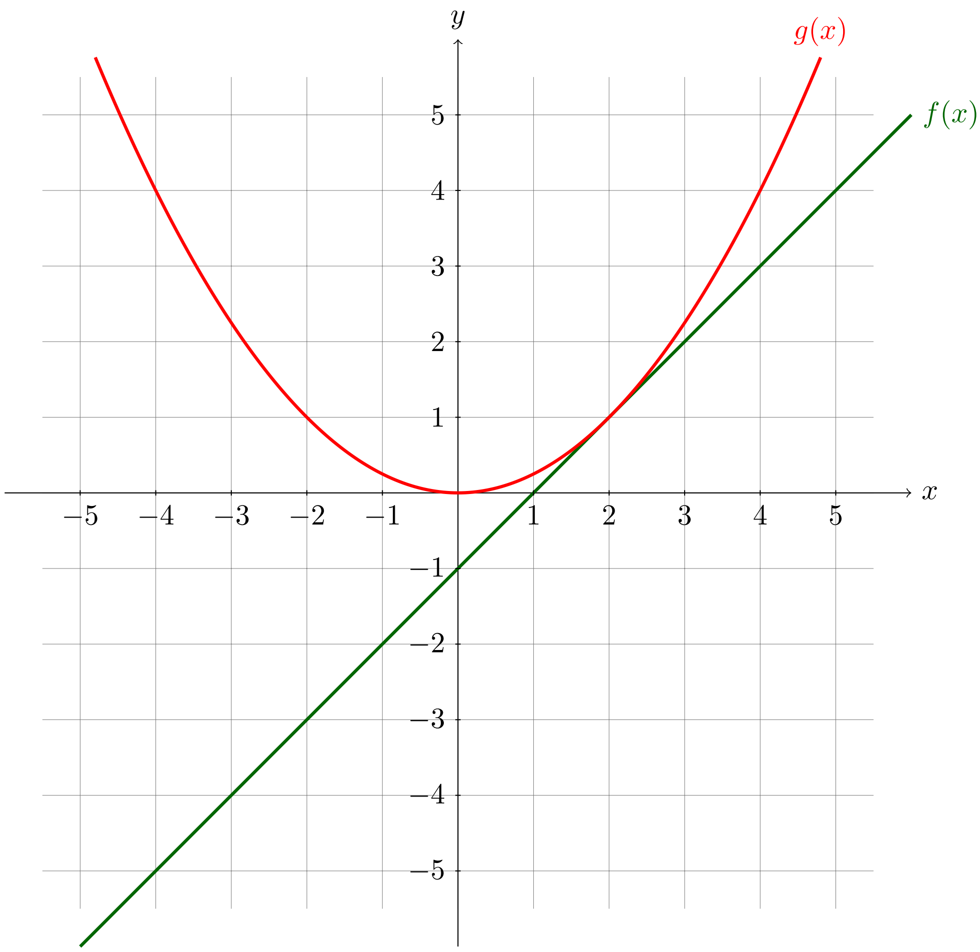


Solutions

General: $f(x) = Cx + C^2$ (here $C = 1$),

Particular: $g(x) = -\frac{1}{4}x^2$ (singular solution).

Equation: $y = xy' - y'^2$

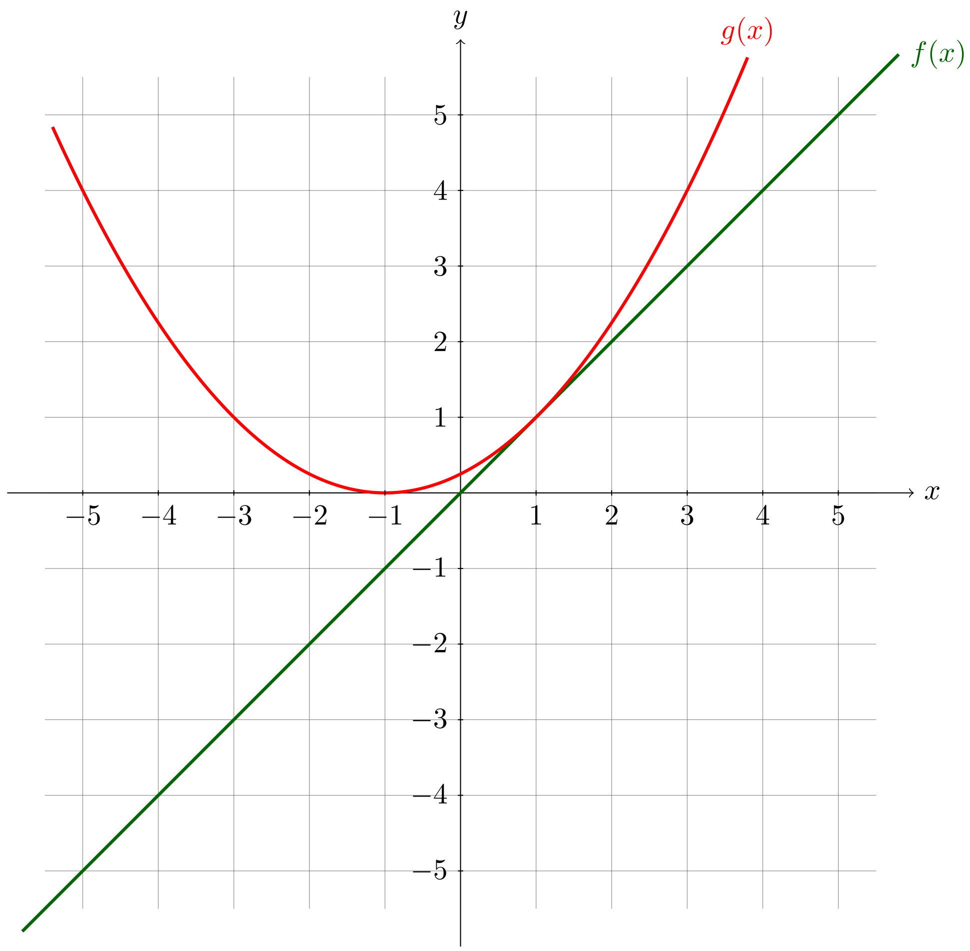


Solutions

General: $f(x) = Cx - C^2$ (here $C = 1$),

Particular: $g(x) = \frac{1}{4}x^2$ (singular solution).

Equation: $y = xy' + y' - y'^2$

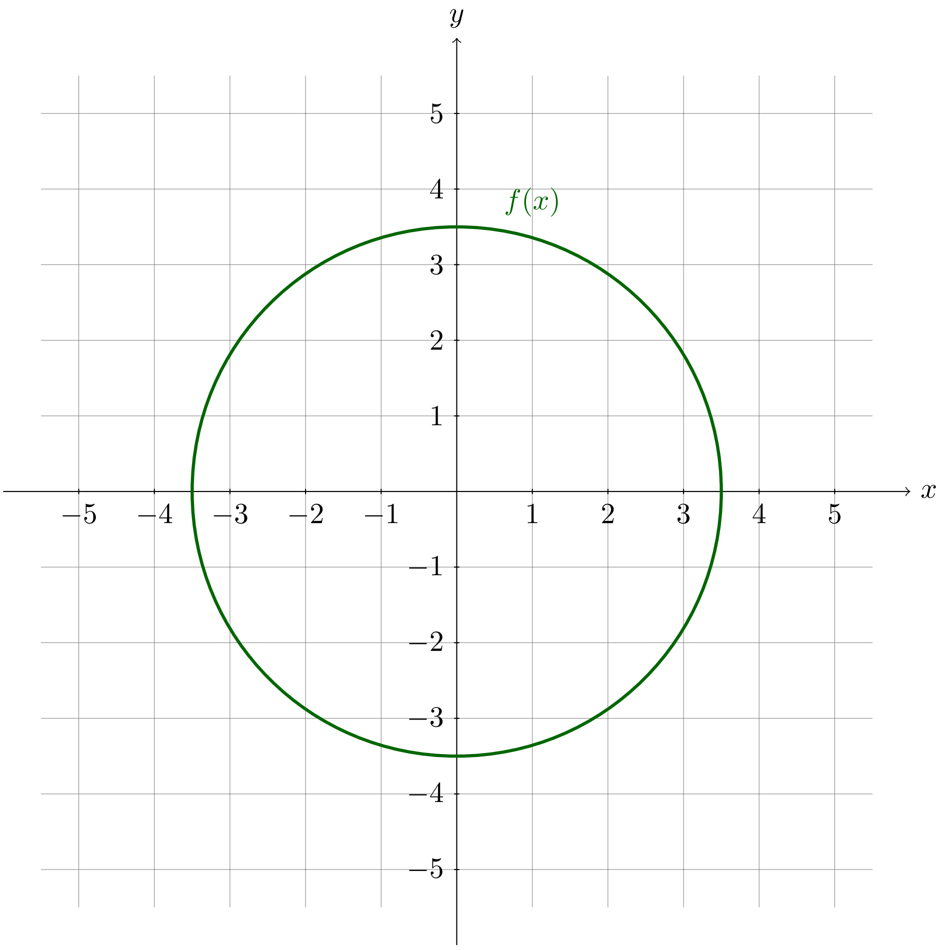


Solutions

General: $f(x) = Cx + C - C^2$ (here $C = 1$),

Particular: $g(x) = \frac{1}{4}(x+1)^2$ (singular solution).

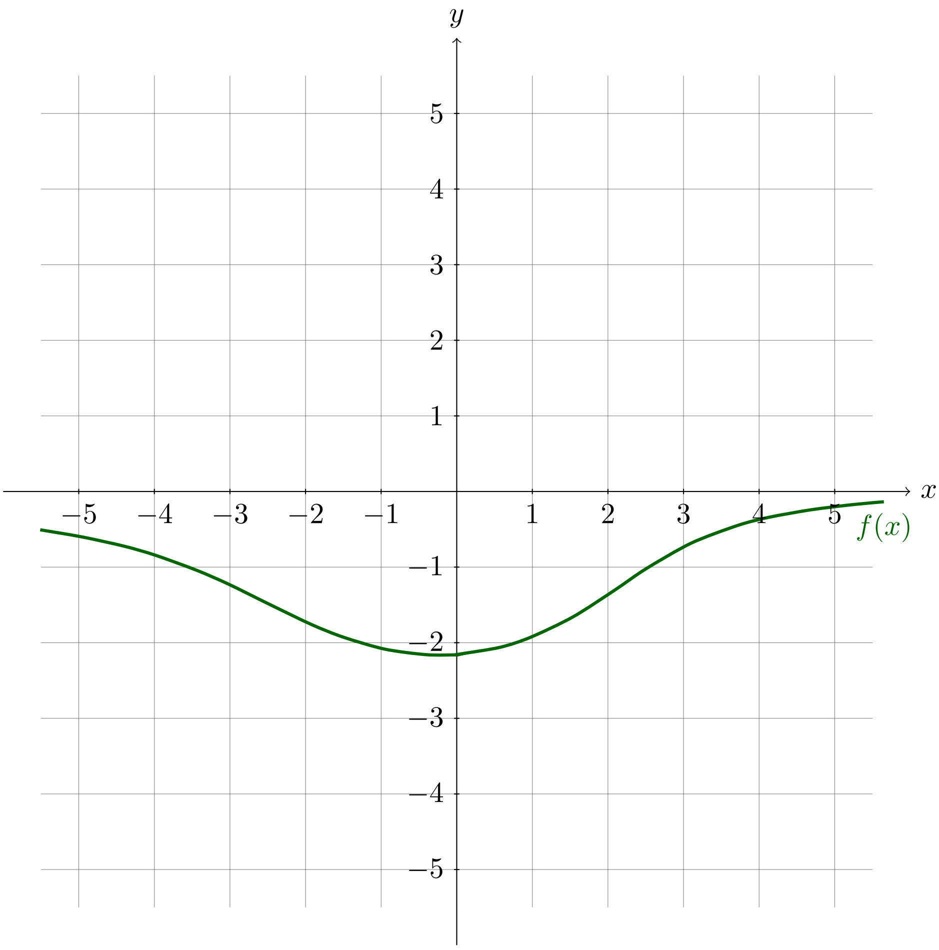
Equation: $(x^3 + xy^2)\mathrm{d}x + (x^2y + y^3)\mathrm{d}y = 0$



Solutions

General: $f(x)$ is such that $x^4 + 2x^2 f(x)^2 + f(x)^4 = C$.

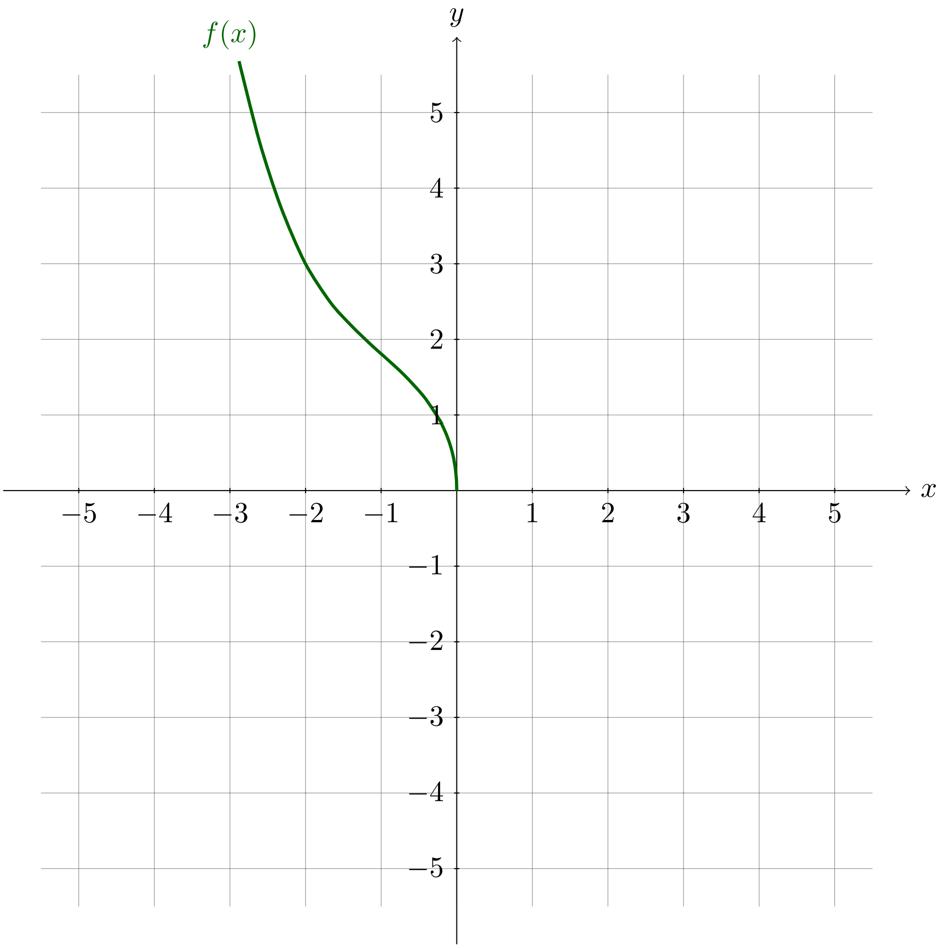
Equation: $(2xy - 1)\mathrm{d}x + (3y^2 + x^2)\mathrm{d}y = 0$



Solutions

General: $f(x)$ is such that $-x + x^2 f(x) + f(x)^3 = C$ (here $C = -10$).

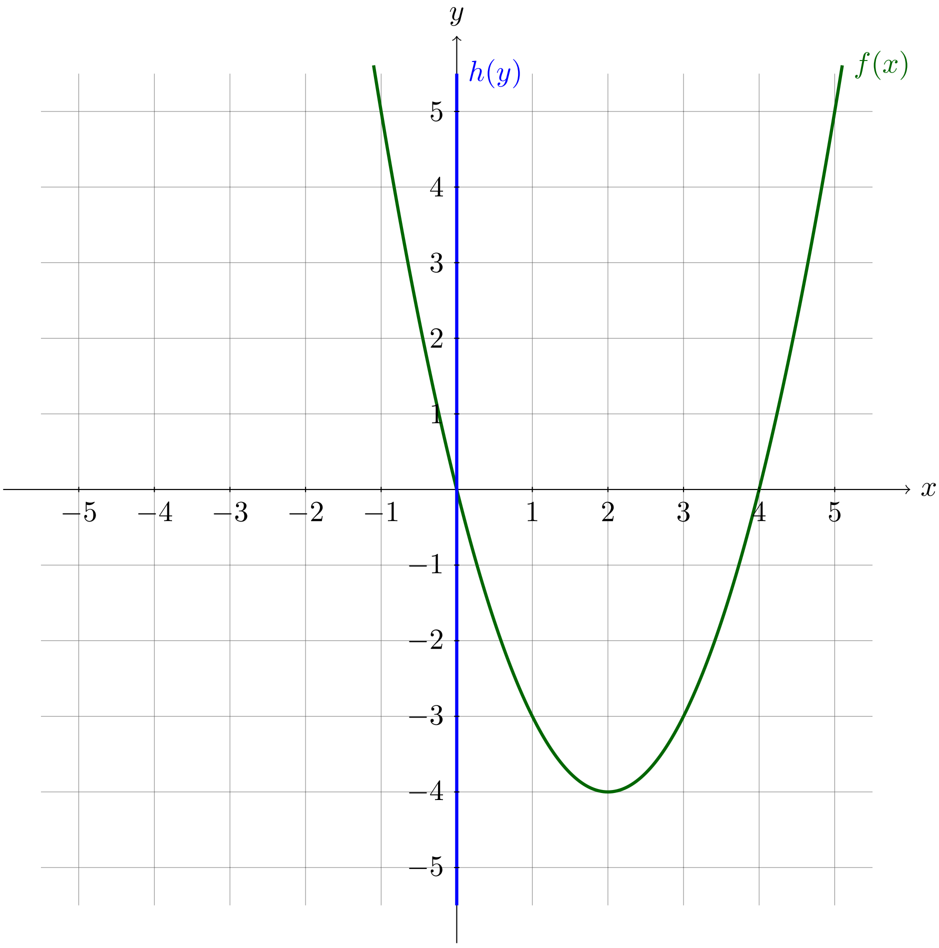
Equation: $\left(\frac{2x}{y} + \frac{y^2}{x^2} + y - 1\right) \mathrm{d}x + \left(-\frac{x^2}{y^2} - \frac{2y}{x} + x + 1\right) \mathrm{d}y = 0$



Solutions

General: $f(x)$ is such that $\frac{x^2}{f(x)} - \frac{f(x)^2}{x} + (x+1)(f(x)-1) = C$.

Equation: $(x^2 + y)\mathrm{d}x - x\mathrm{d}y = 0$

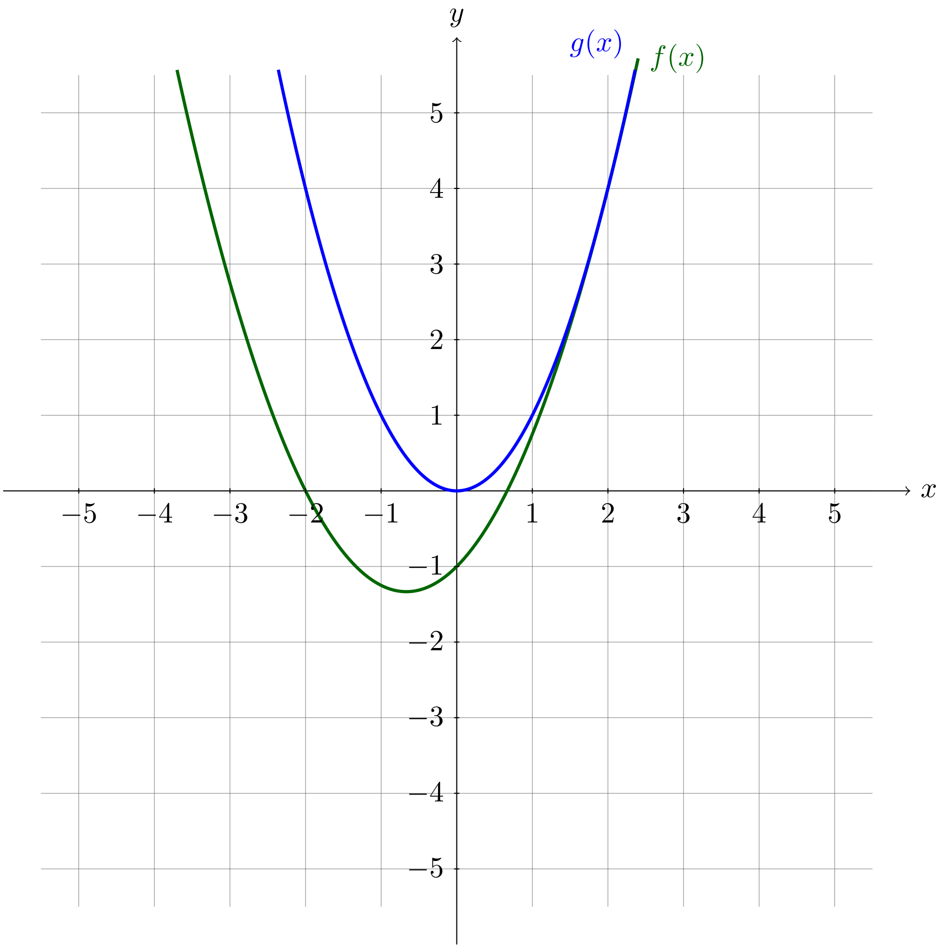


Solutions

General: $f(x) = x^2 - Cx$ (here $C = 4$),

Particular: $h(y) = 0$ for $y \neq 0$.

Equation: $\left(\sqrt{x^2 - y} + 2x\right) \mathrm{d}x - \mathrm{d}y = 0$

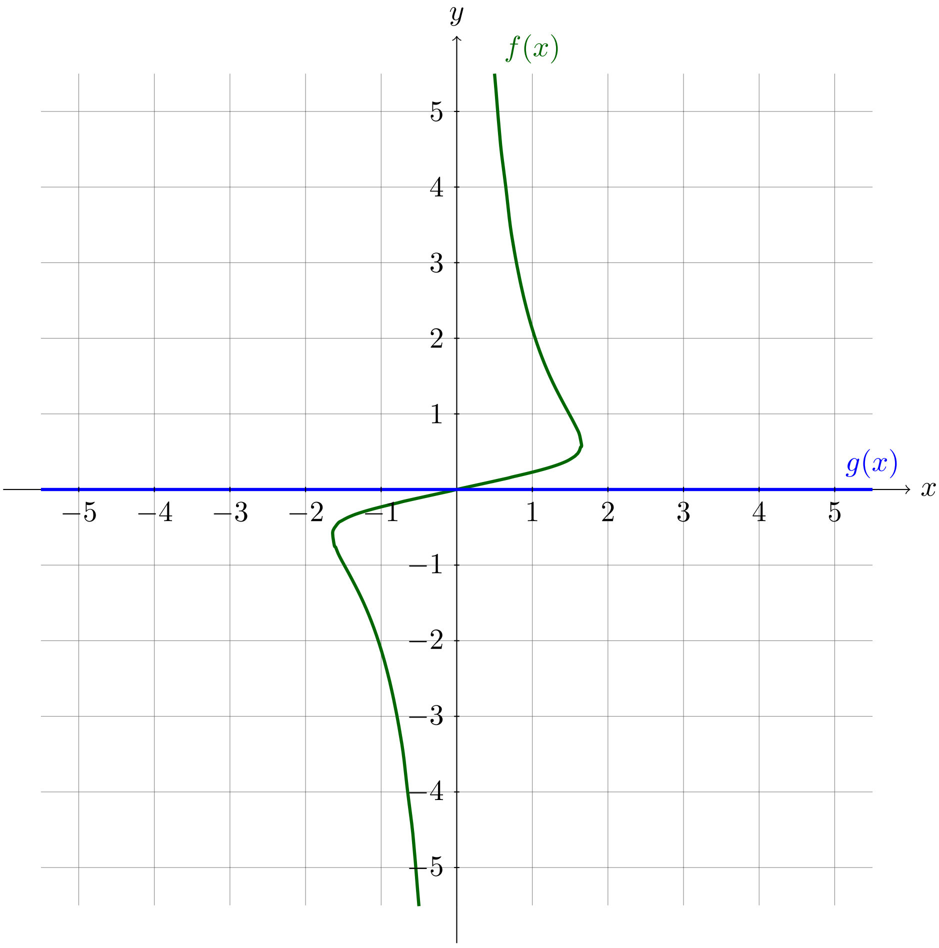


Solutions

General: $f(x) = \frac{3}{4}x^2 + Cx - C^2$ (here $C = 1$),

Particular: $g(x) = x^2$ (singular solution).

Equation: $x^2y^3 + y + (x^3y^2 - x)y' = 0$

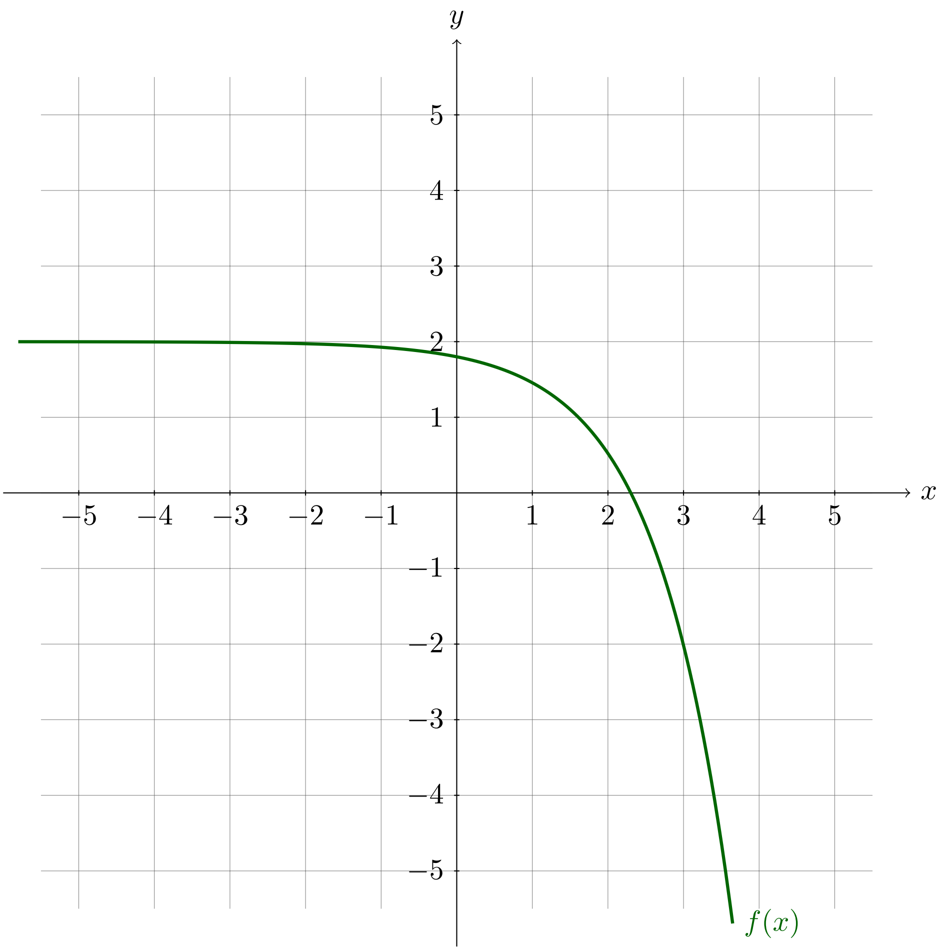


Solutions

General: $f(x)$ is such that $x^2 f(x)^2 + 2\ln\frac{x}{f(x)} = C$ (here $C = 3$),

Particular: $g(x) = 0$ for $x \neq 0$.

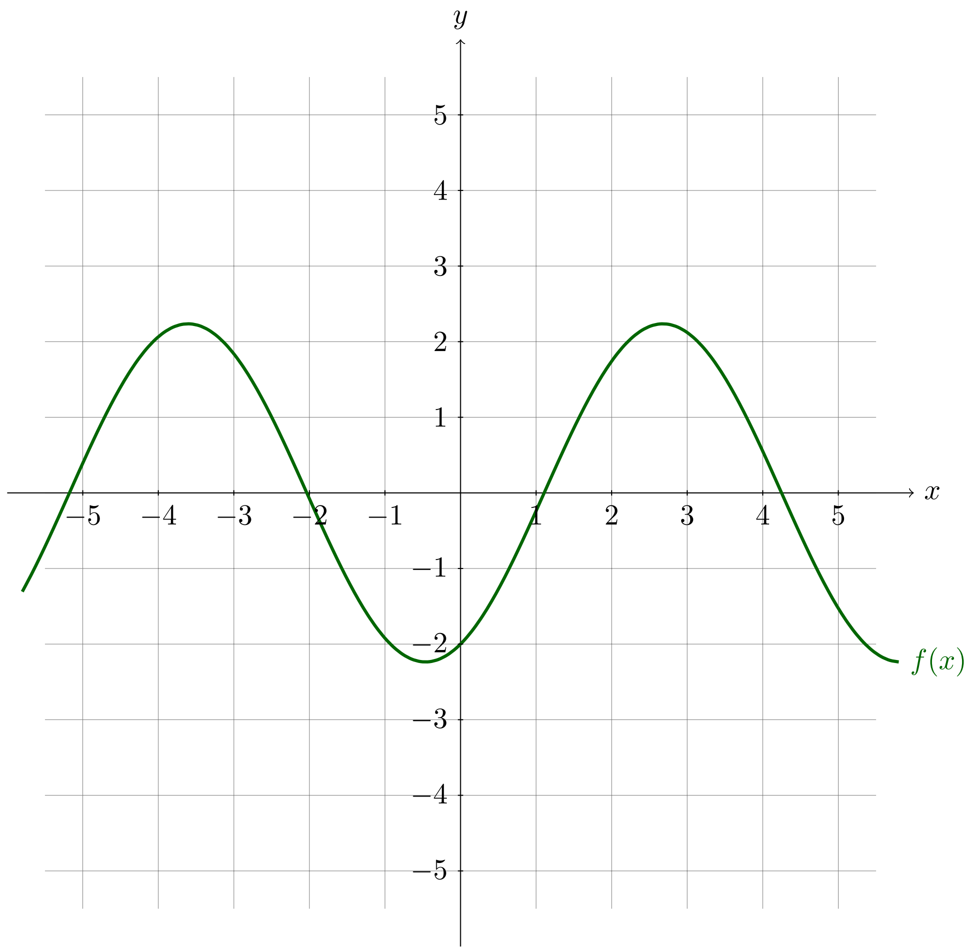
Equation: $y'' - y' = 0$



Solutions

General: $f(x) = C_1 + C_2 e^x$, where $C_i \in \mathbb{R}$ for $i = 1, 2$
(here $C_1 = 2,\ C_2 = \frac{1}{5}$).

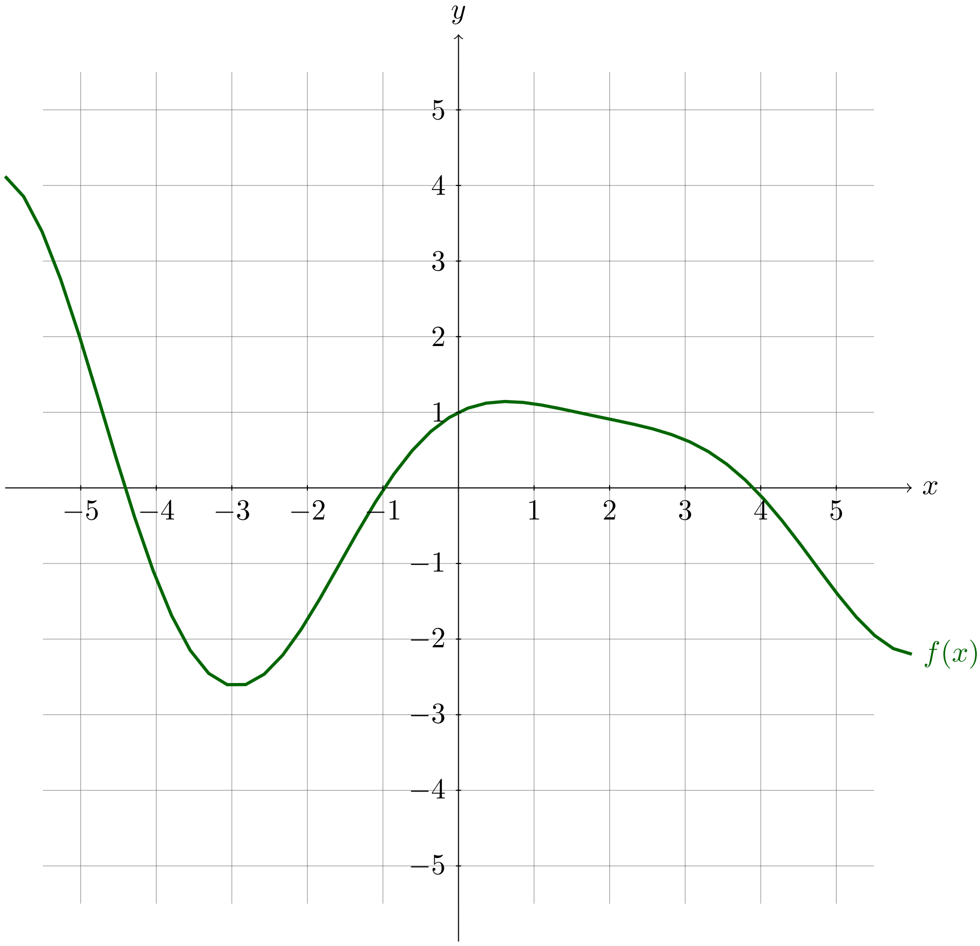
Equation: $y'' + y = 0$



Solutions

General: $f(x) = C_1 \sin x + C_2 \cos x$, where $C_i \in \mathbb{R}$ for $i = 1, 2$
(here $C_1 = 1,\ C_2 = -2$).

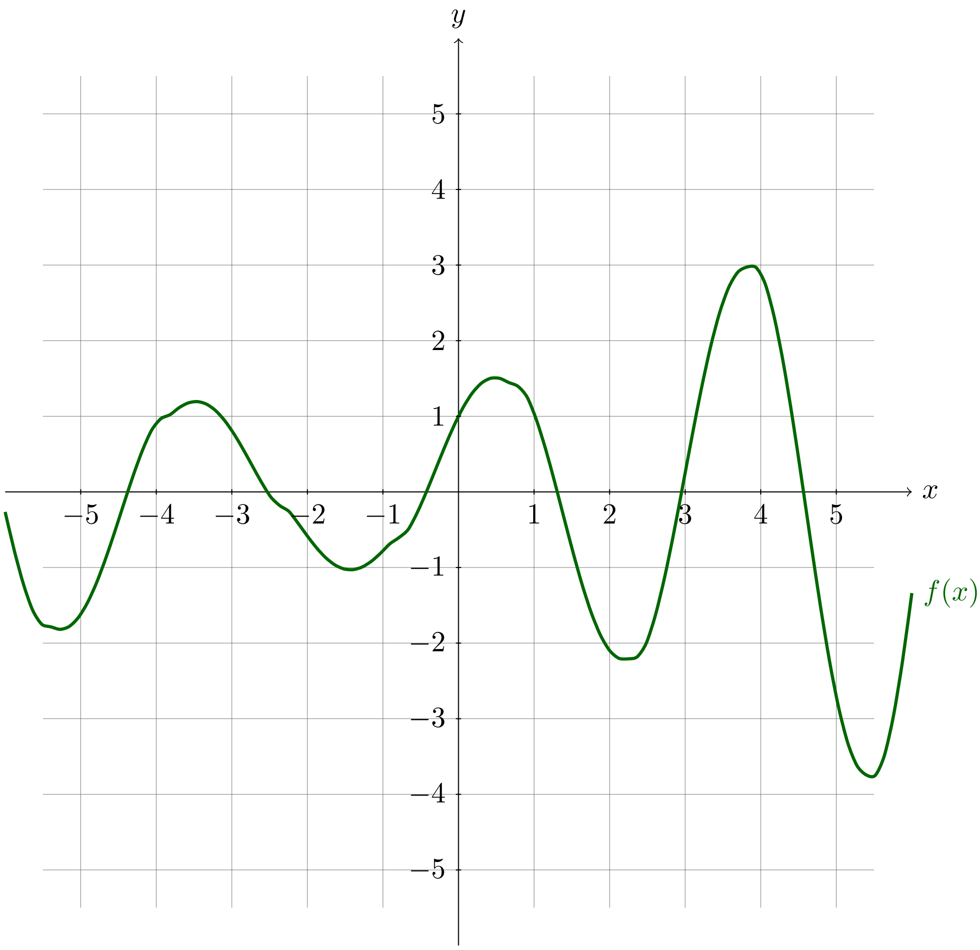
Equation: $y'' + y = \sin x + \cos 2x$



Solutions

General: $f(x) = -\frac{1}{2}x\cos x - \frac{1}{3}\cos 2x + C_1\cos x + C_2\sin x$, where $C_i \in \mathbb{R}$ for $i = 1, 2$, (here $C_1 = C_2 = 1$).

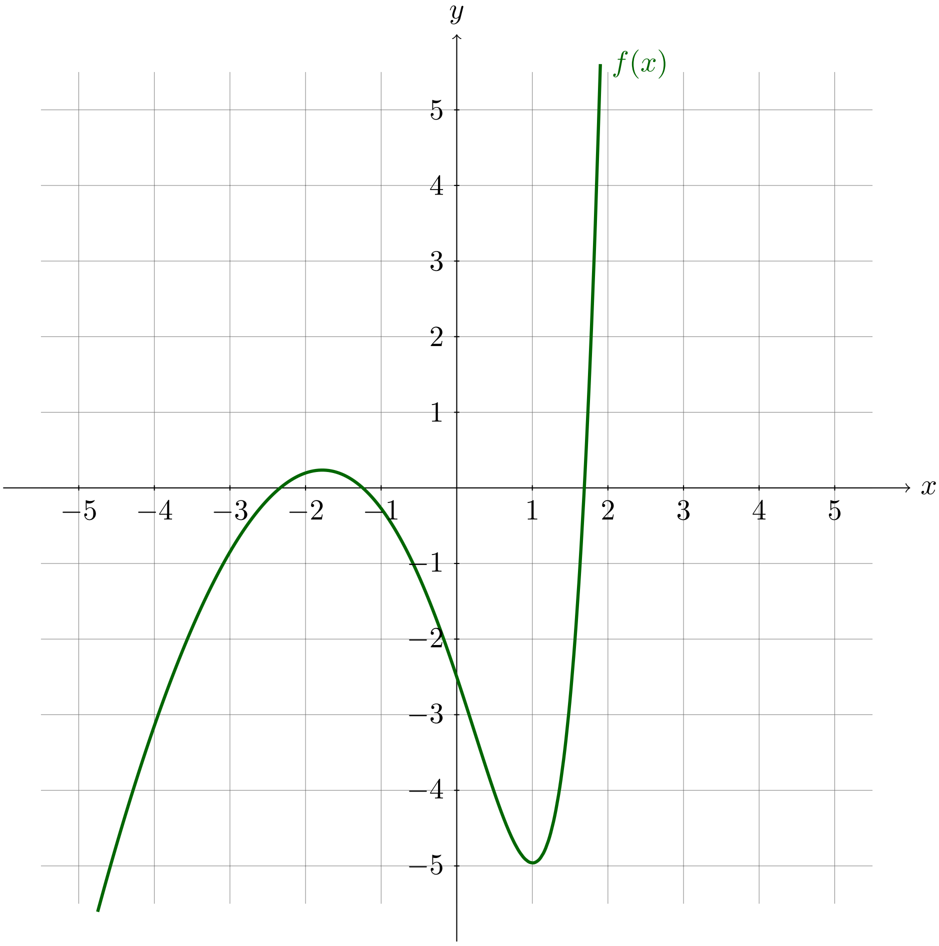
Equation: $y'' + 4y = \dfrac{1}{\cos 2x}$



Solutions

General: $f(x) = C_1 \cos 2x + C_2 \sin 2x + \frac{1}{4}(\ln|\cos 2x|) \cos 2x + \frac{1}{2} x \sin 2x$, where $C_i \in \mathbb{R}$ for $i = 1, 2$, (here $C_1 = 1,\ C_2 = 1$).

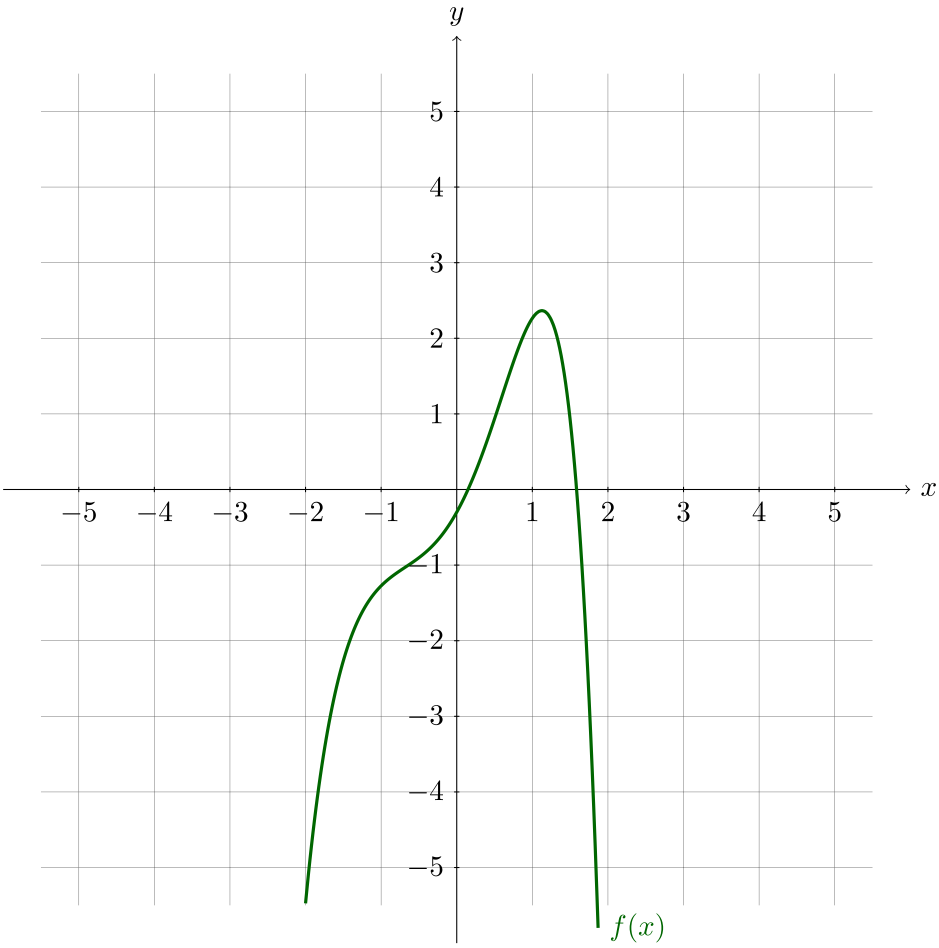
Equation: $y'' - y' = e^x + e^{2x} + x$



Solutions

General: $f(x) = xe^x + \frac{1}{2}e^{2x} - x\left(\frac{1}{2}x + 1\right) + C_1 + C_2e^x$,
where $C_i \in \mathbb{R}$ for $i = 1, 2$ (here $C_1 = 1,\ C_2 = -4$).

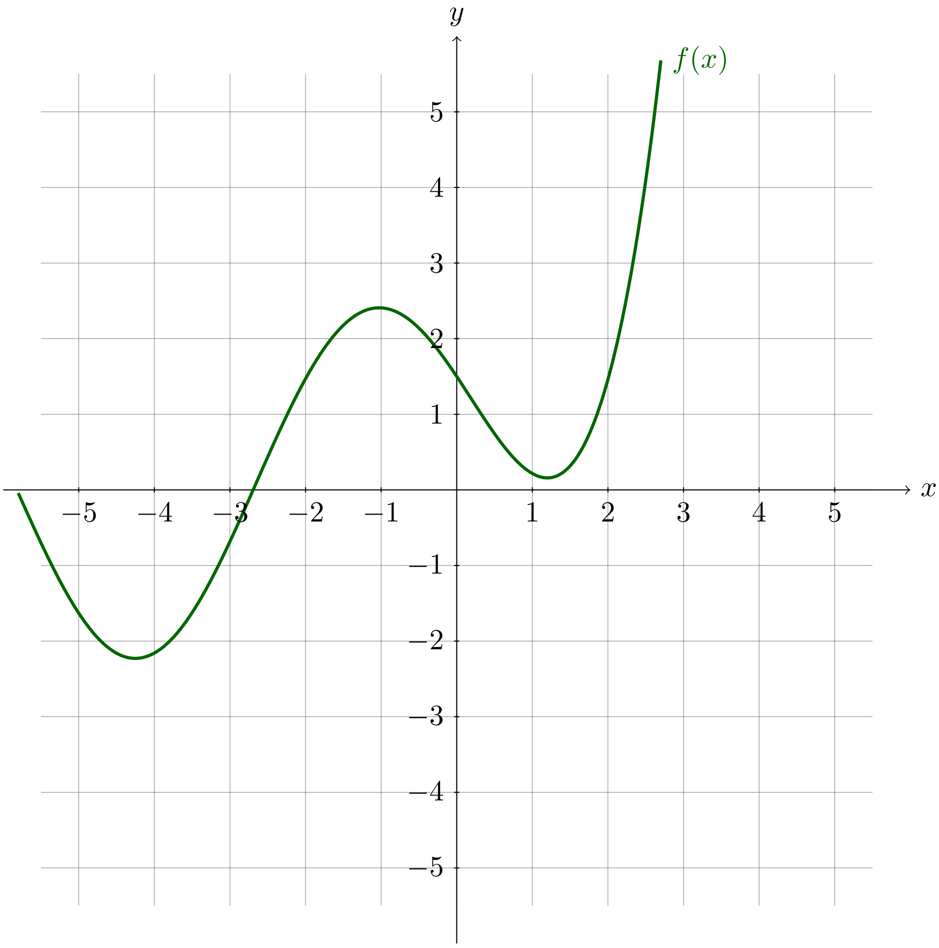
Equation: $y'' - 4y = e^x\left[(-4x+4)\cos x - (2x+6)\sin x\right]$



Solutions

General: $f(x) = e^x\left(x\cos x + \sin x\right) + C_1 e^{2x} + C_2 e^{-2x}$,
where $C_i \in \mathbb{R}$ for $i = 1, 2$ (here $C_1 = -\frac{1}{5},\ C_2 = -\frac{1}{10}$).

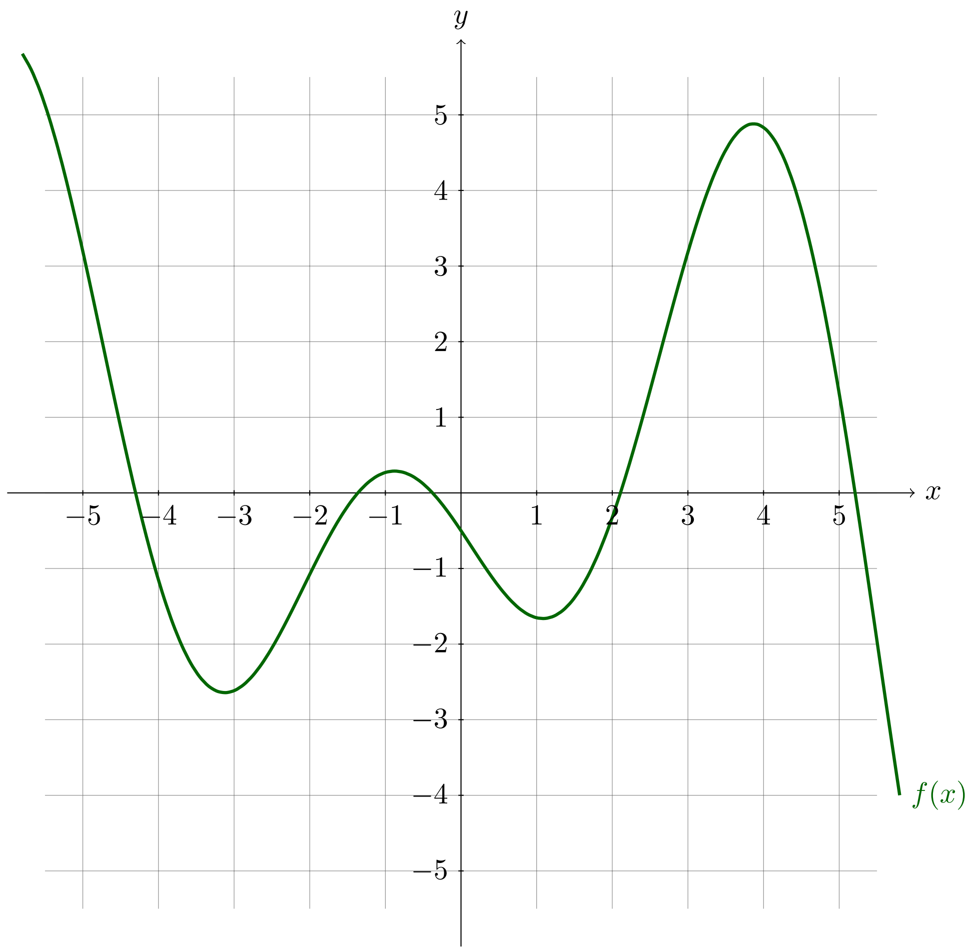
Equation: $y'' + y = e^x$



Solutions

General: $f(x) = C_1 \cos x + C_2 \sin x + \frac{1}{2}e^x$,
where $C_i \in \mathbb{R}$ for $i = 1, 2$ (here $C_1 = 1,\ C_2 = -2$).

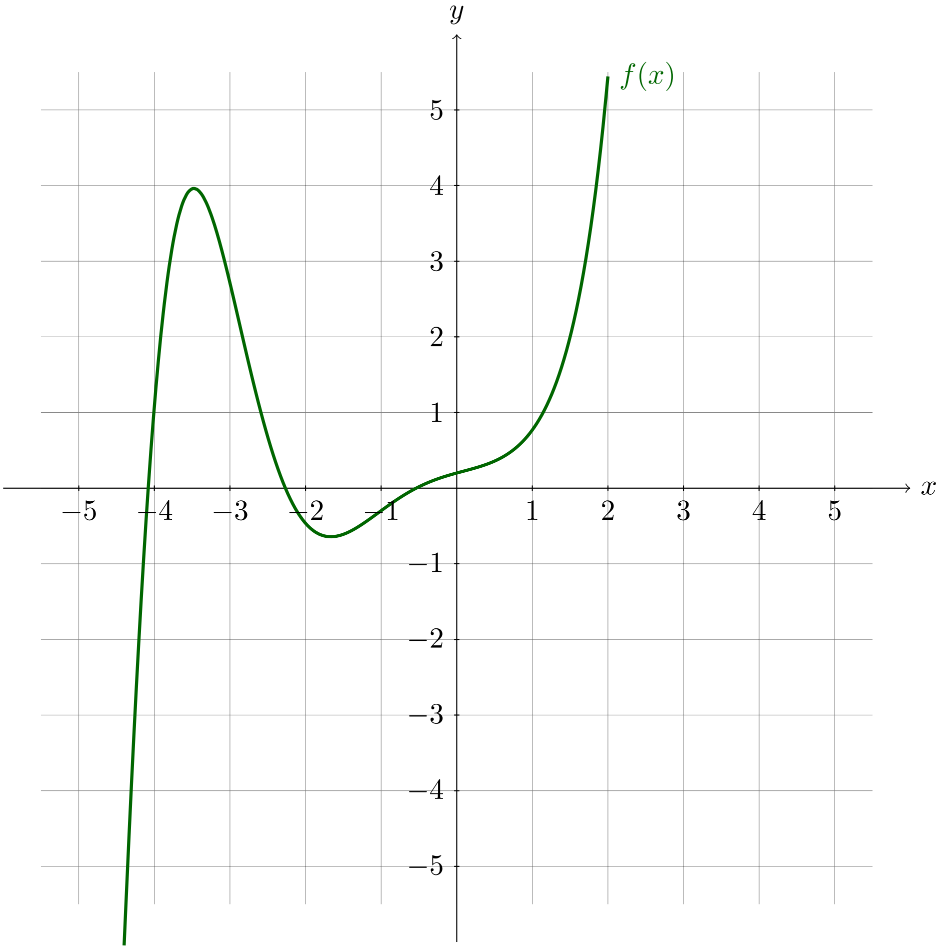
Equation: $y'' + y = 2\sin x - \cos x$



Solutions

General: $f(x) = -x\left(\cos x + \frac{1}{2}\sin x\right) + C_1\cos x + C_2\sin x$,
where $C_i \in \mathbb{R}$ for $i = 1, 2$ (here $C_1 = -\frac{1}{2},\ C_2 = -\frac{1}{2}$).

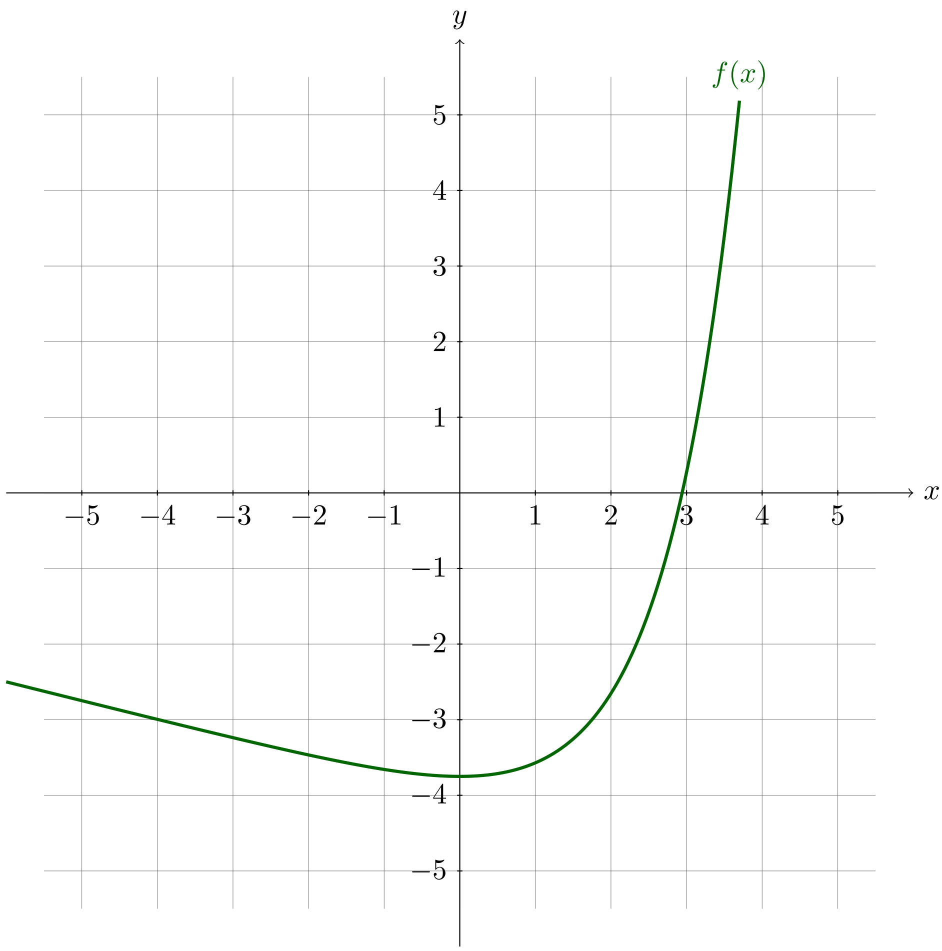
Equation: $y''' - 8y = 0$



Solutions

General: $f(x) = C_1 e^{2x} + e^{-x}\left(C_2 \cos\sqrt{3}x + C_3 \sin\sqrt{3}x\right),$
where $C_i \in \mathbb{R}$ for $i = 1, 2, 3$, (here $C_1 = C_2 = C_3 = \frac{1}{10}|$.

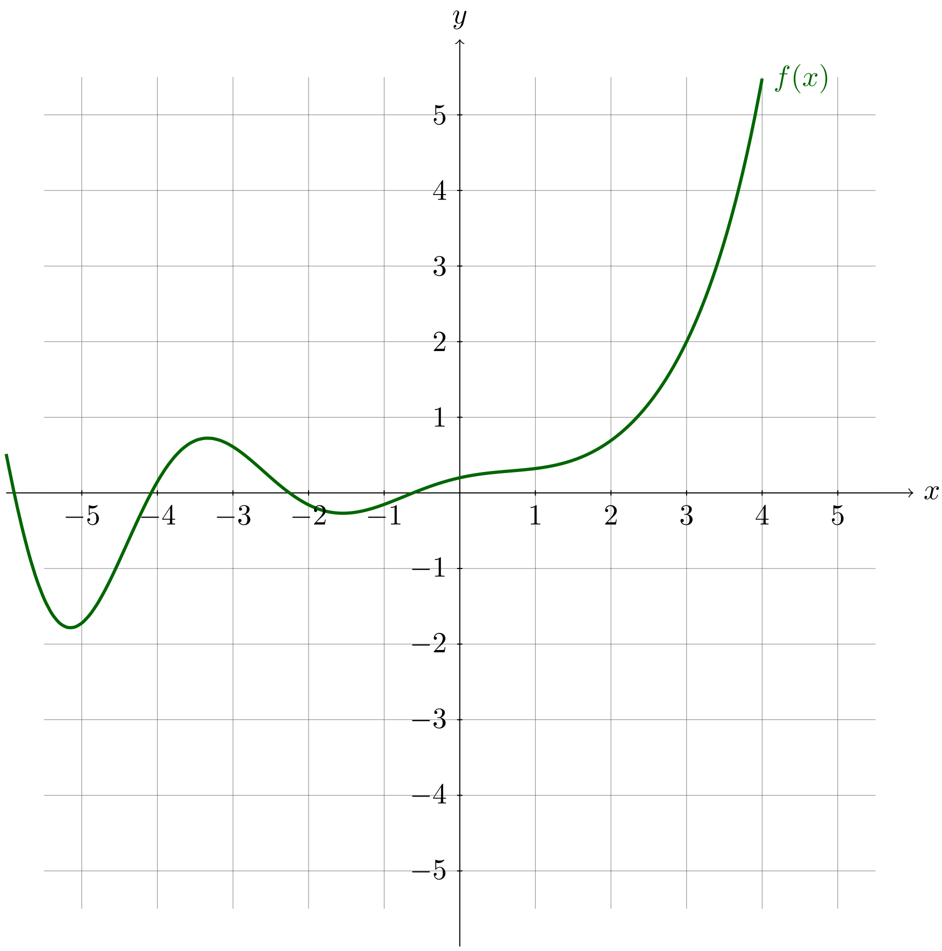
Equation: $y''' - y'' = 0$



Solutions

General: $f(x) = C_1 + C_2 x + C_3 e^x$, where $C_i \in \mathbb{R}$ for $i = 1, 2, 3$, (here $C_1 = -4,\ C_2 = -\frac{1}{4},\ C_3 = \frac{1}{4}$).

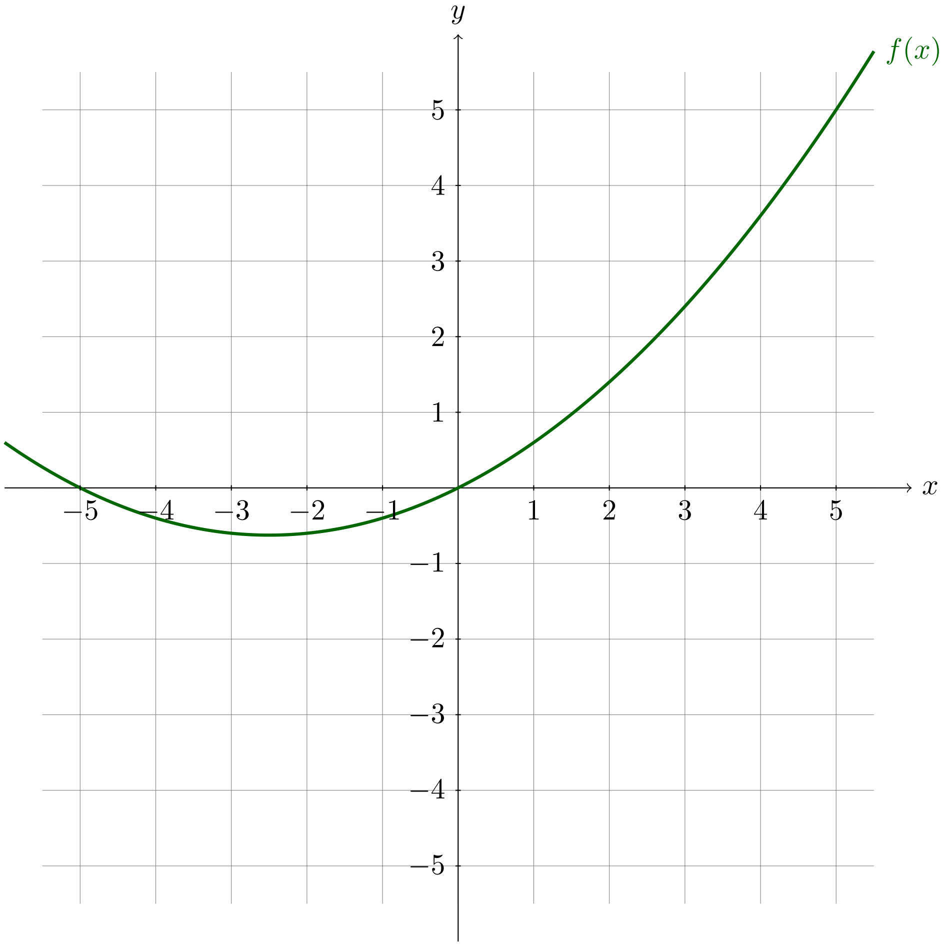
Equation: $y''' + 3y' - 4y = 0$



Solutions

General: $f(x) = C_1 e^x + e^{-\frac{1}{2}x}\left(C_2 \cos\sqrt{\frac{15}{4}}x + C_3 \sin\sqrt{\frac{15}{4}}x\right)$,
where $C_i \in \mathbb{R}$ for $i = 1, 2, 3$, (here $C_1 = C_2 = C_3 = \frac{1}{10}$).

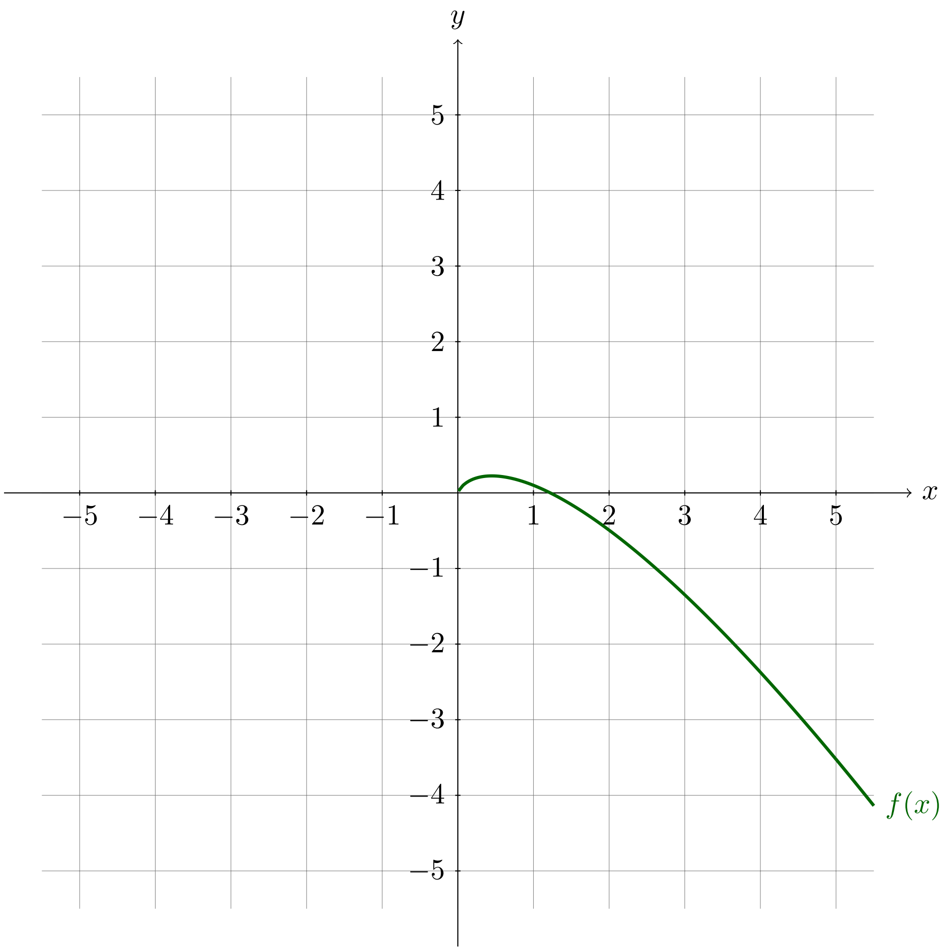
Equation: $x^2y'' - 2xy' + 2y = 0$



Solutions

General: $f(x) = C_1x + C_2x^2$, where $C_i \in \mathbb{R}$ for $i = 1, 2$,
(here $C_1 = \frac{1}{2},\ C_2 = \frac{1}{10}$).

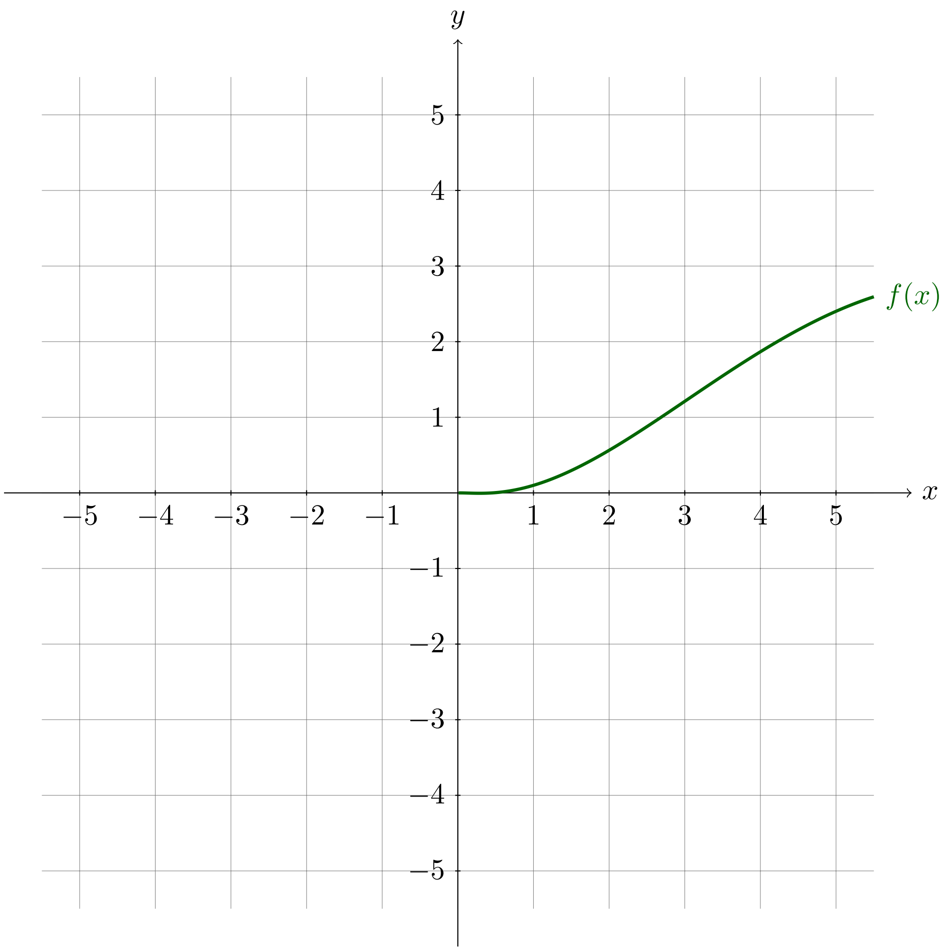
Equation: $x^2y'' - xy' + y = 0$



Solutions

General: $f(x) = C_1x + C_2x \ln x$, where $C_i \in \mathbb{R}$ for $i = 1, 2$,
(here $C_1 = \frac{1}{10},\ C_2 = -\frac{1}{2}$).

Equation: $x^2y'' - 3xy' + 5y = 0$



Solutions

General: $f(x) = x^2\,(C_1 \cos \ln x + C_2 \sin \ln x)$,
where $C_i \in \mathbb{R}$ for $i = 1, 2$, (here $C_1 = C_2 = \frac{1}{10}$).